Midterm Exam No. 02 (Spring 2019)

PHYS 301: Theoretical Methods in Physics

Date: 2019 Mar 6

1. (**20 points.**) Find the eigenvalues of

$$\mathbf{A} = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}. \tag{1}$$

2. (**20 points.**) The Pauli matrix

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{2}$$

is written in the eigenbasis of

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

Write σ_y in the eigenbasis of

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{4}$$

3. (**20 points.**) Using the property of Kronecker δ-function and Levi-Civita symbol evaluate

$$\varepsilon_{ijm}\delta_{mn}\varepsilon_{nij}. \tag{5}$$

4. (**20 points.**) Evaluate

$$\boldsymbol{\nabla} r, \tag{6}$$

where $r = \sqrt{x^2 + y^2 + z^2}$.

5. (**20 points.**) The Pauli matrices are traceless Hermitian matrices that satisfy

$$\sigma_i\sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k, \tag{7}$$

where i, j, are either $1, 2$, or 3. Evaluate

$$\Big[\sigma_i, \big[\sigma_j, \sigma_k\big]\Big]. \tag{8}$$