Midterm Exam No. 03 (Spring 2019)

PHYS 301: Theoretical Methods in Physics

Date: 2019 Apr 12

1. (**20 points.**) Evaluate
$$\boldsymbol{\nabla} r, \tag{1}$$
where r is the magnitude of position coordinate $\mathbf{r}$ in three dimensions and $\boldsymbol{\nabla}$ is the gradient operator in three dimensions. Express your result in simplified form.

2. (**20 points.**) Evaluate the left hand side of
$$\boldsymbol{\nabla} \times \left(\frac{\hat{\mathbf{z}}}{\rho} \right) = \hat{\mathbf{z}}\, f + \hat{\boldsymbol{\rho}}\, g + \hat{\boldsymbol{\phi}}\, h. \tag{2}$$
Thus, determine f, g, and h. Here (z, ρ, ϕ) are cylindrical polar coordinates.

3. (**20 points.**) Evaluate the integral
$$\int_{-1}^{1} dx\, 9x^2 \delta(3x + 1). \tag{3}$$

4. (**20 points.**) Evaluate
$$\int_V d^3 r \boldsymbol{\nabla} \cdot \left(\frac{\hat{\mathbf{r}}}{r^2} \right), \tag{4}$$
where the volume V is the volume of a sphere of radius R.

5. (**20 points.**) A critically damped harmonic oscillator is described by the differential equation
$$\left[\frac{d^2}{dt^2} + 2\omega_0 \frac{d}{dt} + \omega_0^2 \right] x(t) = 0, \tag{5}$$
where ω_0 is a characteristic frequency. Find the solution $x(t)$ for intial conditions $x(0) = x_0$ and $\dot{x}(0) = 0$. Plot $x(t)$ as a function of t in the following graph where $x_0\, e^{-\omega_0 t}$ is already plotted for reference. For what t is the solution $x(t)$ a maximum?

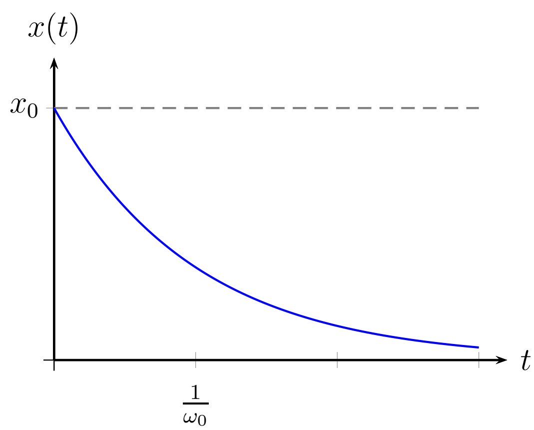

Figure 1: Critically damped harmonic oscillator.