Homework No. 01 (2019 Spring)

PHYS 301: Theoretical Methods in Physics

Due date: Wednesday, 2019 Jan 23, 9:00 AM, in class

1. (**10 points.**) Find the real and imaginary part of

$$z = \frac{(a+ib)}{(c+id)}. \tag{1}$$

Thus, express z in the form $z = u + iv$. Assume a, b, c, and d are real.

2. (**10 points.**) (Refer Arfken) The complex quantities

$$a = u + iv, \tag{2a}$$
$$b = x + iy \tag{2b}$$

may also be represented as two-dimensional vectors

$$\mathbf{a} = \hat{\mathbf{x}}\, u + \hat{\mathbf{y}}\, v, \tag{3a}$$
$$\mathbf{b} = \hat{\mathbf{x}}\, x + \hat{\mathbf{y}}\, y. \tag{3b}$$

Show that

$$(a^*)b = \mathbf{a}\cdot\mathbf{b} + \hat{\mathbf{z}}\cdot\mathbf{a}\times\mathbf{b}. \tag{4}$$

3. (**30 points.**) Find the fifth roots of unity by solving the equation

$$z^5 = 1. \tag{5}$$

Mark the the points corresponding to the five roots on the complex plane. Find the five roots of the equation

$$z^5 = -1. \tag{6}$$

Mark the roots on the complex plane. Next, find the roots of the equation

$$z^5 = i \tag{7}$$

and mark the roots on the complex plane. Repeat the exercise for $z^5 = -i$. How do these roots match with the fifth roots of unity?