

Homework No. 02 (2019 Spring)

PHYS 301: Theoretical Methods in Physics

Due date: Monday, 2019 Jan 28, 9:00 AM, in class

1. **(100 points.)** Hyperbolic cosine and sine are defined in terms of the exponential function,

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad (1a)$$

$$\sinh x = \frac{e^x - e^{-x}}{2}. \quad (1b)$$

In the following assume x and y to be real.

- (a) Plot $\cosh x$ and $\sinh x$ as functions of x .
- (b) Derive the following identities:

$$\cosh^2 x - \sinh^2 x = 1, \quad (2a)$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y, \quad (2b)$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y, \quad (2c)$$

$$\frac{d}{dx} \cosh x = \sinh x, \quad (2d)$$

$$\frac{d}{dx} \sinh x = \cosh x, \quad (2e)$$

$$\int dx \cosh x = \sinh x, \quad (2f)$$

$$\int dx \sinh x = \cosh x. \quad (2g)$$

- (c) Show that

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}). \quad (2h)$$

Derive the corresponding expression for $\cosh^{-1} x$.

2. **(20 points.)** Find all z that satisfies the equation

$$e^z = e^{iz}. \quad (3)$$