Homework No. 05 (2019 Spring)

PHYS 301: Theoretical Methods in Physics

Due date: Wednesday, 2019 Feb 13, 9:00 AM, in class

1. (**45 points.**) A particular representation of Pauli matrices is

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{1}$$

(In particular, these are Pauli matrices in the eigenbasis of σ_z.) Find the eigenvalues, normalized eigenvectors, and diagonalizing matrix, for each of the three Pauli matrix. Verify that your results satisfy the eigenvalue equation.

2. (**20 points.**) Construct the matrix

$$\boldsymbol{\sigma} \cdot \hat{\mathbf{r}}, \tag{2}$$

where

$$\boldsymbol{\sigma} = \sigma_x \hat{\mathbf{i}} + \sigma_y \hat{\mathbf{j}} + \sigma_z \hat{\mathbf{k}}, \tag{3}$$

$$\hat{\mathbf{r}} = \sin\theta\cos\phi\hat{\mathbf{i}} + \sin\theta\sin\phi\hat{\mathbf{j}} + \cos\theta\hat{\mathbf{k}}. \tag{4}$$

Use the representation of Pauli matrices is

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{5}$$

Find the eigenvalues of the matrix $\boldsymbol{\sigma} \cdot \hat{\mathbf{r}}$.

3. (**20 points.**) The Pauli matrix

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{6}$$

is written in the eigenbasis of

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{7}$$

Write σ_x in the eigenbasis of

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{8}$$

Note that this representation has the arbitraryness of the choice of phase in the eigenvectors.