Homework No. 07 (2019 Spring)

PHYS 301: Theoretical Methods in Physics

Due date: Wednesday, 2019 Mar 6, 9:00 AM, in class

1. (**20 points.**) Derive the following vector identities (using index notation)

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B}), \tag{1a}$$

$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C}), \tag{1b}$$

2. (**20 points.**) Verify the following identities:

$$\boldsymbol{\nabla} r = \frac{\mathbf{r}}{r} = \hat{\mathbf{r}}, \tag{2a}$$

$$\boldsymbol{\nabla} \mathbf{r} = \mathbf{1}. \tag{2b}$$

Further, show that

$$\boldsymbol{\nabla} \cdot \mathbf{r} = 3, \tag{3a}$$

$$\boldsymbol{\nabla} \times \mathbf{r} = 0. \tag{3b}$$

Here r is the magnitude of the position vector $\mathbf{r}$, and $\hat{\mathbf{r}}$ is the unit vector pointing in the direction of $\mathbf{r}$.