Homework No. 08 (2019 Spring)

PHYS 301: Theoretical Methods in Physics

Due date: Monday, 2019 Mar 25, 9:00 AM, in class

1. (**20 points.**) Given the flow velocity field

$$\mathbf{v} = \omega\rho\hat{\boldsymbol{\phi}} \tag{1}$$

determine the vorticity $\boldsymbol{\nabla} \times \mathbf{v}$ of the flow. Illustrate the flow field and the vorticity using the associated vector field lines. Here ω is a constant, and ρ and ϕ are cylindrical polar coordinates.

2. (**20 points.**) Given the flow velocity field

$$\mathbf{v} = \frac{c}{\rho}\hat{\boldsymbol{\phi}} \tag{2}$$

determine the vorticity $\boldsymbol{\nabla} \times \mathbf{v}$ of the flow. Illustrate the flow field and the vorticity using the associated vector field lines. Here c is a constant, and ρ and ϕ are cylindrical polar coordinates. Let $\rho \neq 0$.

3. (**20 points.**) (Based on problem 1.26 Griffiths 4th edition.)
Calculate the Laplacian of the following functions:

   (a) $T_a = x^2 + 2xy + 3z + 4$

   (b) $T_b = \sin x \sin y \sin z$

   (c) $\mathbf{v} = x^2\,\hat{\mathbf{x}} + 3xz^2\,\hat{\mathbf{y}} - 2xz\,\hat{\mathbf{z}}$