Homework No. 09 (2019 Spring)

PHYS 301: Theoretical Methods in Physics

Due date: Monday, 2019 Apr 1, 9:00 AM, in class

1. (**10 points.**) Check the fundamental theorem of divergence,

$$\int_V d^3x\, \boldsymbol{\nabla} \cdot \mathbf{E} = \oint_S d\mathbf{a} \cdot \mathbf{E}, \tag{1}$$

   for the vector field $\mathbf{E} = x\,\hat{\mathbf{x}}$. Use the volume V to be a cube of length L with an edge of the cube parallel to the x-axis. Using the fact that the divergence of a vector field at a point tells us whether a point is a source or sink of the field, estimate the distribution of the source and sink for the field $\mathbf{E}$?

2. (**10 points.**) Evaluate the flux,

$$\int_S d\mathbf{a} \cdot \mathbf{E}, \tag{2}$$

   of the uniform (homogeneous in space) field

$$\mathbf{E} = E\,\hat{\mathbf{z}} \tag{3}$$

   through a hemispherical bowl of radius R placed such that the circle determining the edge of the hemisphere is on the x-y plane. Show that the result is independent of the position of the center of the circle.

3. (**10 points.**) Check the fundamental theorem of curl,

$$\int_S d\mathbf{a} \cdot \boldsymbol{\nabla} \times \mathbf{E} = \oint_C d\mathbf{l} \cdot \mathbf{E}, \tag{4}$$

   (where the sense of the line integration is given by the right hand rule: the contour C is traversed in the sense of the fingers of the right hand and the thumb points in the sense of the orientation of the surface,) for the vector field $\mathbf{E} = y\,\hat{\mathbf{x}} + z\,\hat{\mathbf{y}} + x\,\hat{\mathbf{z}}$. Use the surface S to be a square of length L on the $z = 0$ plane with one side parallel to the x-axis. Using the fact that the curl of a vector field at a point is a measure of the torque experienced by a (point) dipole at the point, estimate the torque field.

4. (**10 points.**) Evaluate the vector area of a hemispherical bowl of radius R given by

$$\mathbf{a} = \int_S d\mathbf{a}, \tag{5}$$

   where S stands for the surface of the hemispherical bowl. Next, evaluate the above vector area on the surface of a sphere.