Homework No. 11 (2019 Spring)

PHYS 301: Theoretical Methods in Physics

Due date: Monday, 2019 Apr 22, 9:00 AM, in class

1. (**20 points.**) Fourier series (or transformation) is defined as ($0 \le \theta < 2\pi$)

$$f(\theta) = \frac{1}{2\pi} \sum_{-\infty}^{\infty} e^{im\theta} a_m, \tag{1}$$

where the coefficients a_m are determined using

$$a_m = \int_0^{2\pi} d\theta \, e^{-im\theta} f(\theta). \tag{2}$$

Determine the particular function $f(\theta)$ which leads to

$$a_m = 1 \tag{3}$$

for all m. That is, all the Fourier coefficients are contributing equally in the series.

2. (**20 points.**) Fourier series (or transformation) is defined as ($0 \le \theta < 2\pi$)

$$f(\theta) = \frac{1}{2\pi} \sum_{-\infty}^{\infty} e^{im\theta} a_m, \tag{4}$$

where the coefficients a_m are determined using

$$a_m = \int_0^{2\pi} d\theta \, e^{-im\theta} f(\theta). \tag{5}$$

Determine all the Fourier coefficients a_m for

$$f(\theta) = \cos\theta = \frac{(e^{i\theta} + e^{-i\theta})}{2}. \tag{6}$$

3. (**20 points.**) Fourier series (or transformation) is defined as ($-\infty < x < \infty$)

$$f(x) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ikx} a(k), \tag{7}$$

where the coefficients $a(k)$ are determined using

$$a(k) = \int_{-\infty}^{\infty} dx e^{-ikx} f(x). \tag{8}$$

(a) Show that

$$\frac{d^n f(x)}{dx^n} = \int_{-\infty}^{\infty} \frac{dk}{2\pi} (ik)^n e^{ikx} a(k). \tag{9}$$

(b) Show that the differential equation

$$-\left(\frac{d^2}{dx^2} - \omega^2\right) f(x) = \delta(x) \tag{10}$$

in the Fourier space is the algebraic equation

$$(k^2 + \omega^2) a(k) = 1. \tag{11}$$

Thus, the solution to the differential equation is the Fourier transform of

$$a(k) = \frac{1}{\omega^2 + k^2}. \tag{12}$$