Midterm Exam No. 01 (Spring 2019)

PHYS 420: Electricity and Magnetism II

Date: 2019 Feb 8

1. (**20 points.**) A steady current I flows down an infinitely long cylindrical wire of radius a. Using Ampère's law find the magnetic field, both inside and outside the wire, if the current is uniformly distributed over the outside surface of the wire.

2. (**20 points.**) The magnetic field at a distance R from a wire of infinite extent carrying a steady current I is given by

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{2I}{R}\hat{\boldsymbol{\phi}}, \tag{1}$$

where the direction of $\hat{\boldsymbol{\phi}}$ is given by the right-hand rule. Find the magnetic field at point o in Fig. 2 in terms of distances a and b and current I.

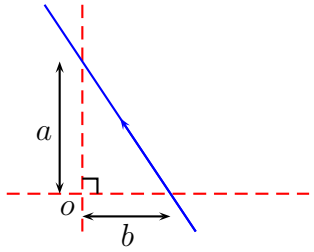

Figure 1: Problem 2

3. (**20 points.**) The magnetic field for a straight wire of infinite extent carrying a steady current I is given by

$$\mathbf{B}(\mathbf{r}) = \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I}{2\pi\rho}. \tag{2}$$

Verify that $\boldsymbol{\nabla}\cdot\mathbf{B} = 0$ everywhere. In particular, investigate if the magnetic field is divergenceless on the wire, where $\rho = 0$.

4. (**20 points.**) The magnetic field $\mathbf{B}(\mathbf{r})$ is given in terms of the magnetic vector potential $\mathbf{A}(\mathbf{r})$ by the relation

$$\mathbf{B} = \boldsymbol{\nabla}\times\mathbf{A}. \tag{3}$$

Find a magnetic vector potential (up to a gauge) for the uniform magnetic field

$$\mathbf{B} = B\,\hat{\mathbf{z}}. \tag{4}$$

Then, find another solution for $\mathbf{A}$ (up to a gauge) that is different from your original solution by more than just a constant. If you designed an experiment to measure $\mathbf{A}$, which one of your solution will the experiment measure?