Midterm Exam No. 02 (Spring 2019)

PHYS 420: Electricity and Magnetism II

Date: 2019 Mar 6

1. (**20 points.**) For an arbitrary function $f(x)$ you are given

$$\int_0^{2\pi} d\phi f(\cos\phi) = a \int_0^{\pi} d\phi f(-\cos\phi). \tag{1}$$

   Find a for which the above equation is an identity..

2. (**20 points.**) The expression for the magnetic vector potential $\mathbf{A}$ and the magnetic field $\mathbf{B}$ for a circular loop of radius a carrying a current I is given in terms of the complete elliptic integrals. An approximate expression for the magnetic vector potential close to the axis is

$$\mathbf{A}(\mathbf{r}) = \hat{\boldsymbol{\phi}}\, A(\rho, z) = \hat{\boldsymbol{\phi}}\, \frac{\mu_0 I}{4\pi} \frac{a^2 \pi \rho}{[z^2 + (\rho + a)^2]^{\frac{3}{2}}}. \tag{2}$$

   Check that $\mathbf{A} = 0$ on the axis. The magnetic field close to the axis, then, is calculated using

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}. \tag{3}$$

   Express the magnetic field close to the axis in the form

$$\mathbf{B}(\mathbf{r}) = -\hat{\boldsymbol{\rho}} \frac{\partial A}{\partial z} + \hat{\mathbf{z}} \left(\frac{\partial A}{\partial \rho} + C \right). \tag{4}$$

   Find C.

3. (**20 points.**) Show that the perimeter of an ellipse, characterized by the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \tag{5}$$

   with eccentricity

$$e = \sqrt{1 - \frac{b^2}{a^2}}, \tag{6}$$

   is given by

$$C = 4aE(e), \tag{7}$$

   where $E(k)$ is the complete elliptic integral of the second kind,

$$E(k) = \int_0^{\frac{\pi}{2}} d\psi \sqrt{1 - k^2 \sin^2 \psi}. \tag{8}$$

4. (**20 points.**) The magnetic field due to an infinitely long solenoid is independent of the radius of the solenoid. In particular, it is uniform inside the solenoid and zero outside. Imagine the limiting case when the radius of the solenoid goes to zero. The magnetic vector potential for such an infinitely thin and infinitely long solenoid is given by

$$\mathbf{A}(\mathbf{r}) = \hat{\boldsymbol{\phi}} \frac{\mu_0}{4\pi} \frac{2\lambda}{\rho}, \tag{9}$$

where λ is the magnetic moment per unit length of the solenoid. Here ρ and ϕ are the cylindrical polar coordinates. The associated magnetic field is given by

$$\mathbf{B} = \hat{\mathbf{z}}\, \mu_0 \lambda\, \delta^{(2)}(\boldsymbol{\rho}). \tag{10}$$

Evaluate

$$\boldsymbol{\nabla} \cdot \mathbf{B}. \tag{11}$$

Is it zero? Illustrate the vector expression for the magnetic vector potential **A** and the magnetic field **B** above using a diagram. A positive charge is moving with velocity **v** in the vicinity of this solenoid. Will the charge sense the presence of the solenoid due to electromagnetic interactions?

5. (**20 points.**) Repeat Problem 4 at home. Use your favorite resource to understand Aharonov-Bohm effect in this context. Submit a brief summary of your understanding of the concept.