

Homework No. 01 (2019 Spring)

PHYS 420: Electricity and Magnetism II

Due date: Wednesday, 2019 Jan 23, 2:00 PM, in class

1. (40 points.) A particle of mass m and charge q moving in a uniform magnetic field \mathbf{B} experiences a velocity dependent force \mathbf{F} given by the expression

$$m \frac{d\mathbf{v}}{dt} = q\mathbf{v} \times \mathbf{B}, \quad (1)$$

where $\mathbf{v}(t) = d\mathbf{x}/dt$ is the velocity of the particle in terms of its position $\mathbf{x}(t)$. Choose the magnetic field to be along the positive z direction, $\mathbf{B} = B\hat{\mathbf{z}}$.

- (a) For the case when the particle is at the origin at time $t = 0$, use the initial conditions

$$\mathbf{v}(0) = 0\hat{\mathbf{x}} + v_0\hat{\mathbf{y}} + 0\hat{\mathbf{z}}, \quad (2)$$

$$\mathbf{x}(0) = 0\hat{\mathbf{x}} + 0\hat{\mathbf{y}} + 0\hat{\mathbf{z}}, \quad (3)$$

to solve the differential equation in Eq. (1) to find the position $\mathbf{x}(t)$ and velocity $\mathbf{v}(t)$ as a function of time. Use $\omega = qB/m$.

- (b) In particular, prove that the particle takes a circular path. What is the radius of the circle? Determine the coordinates of the center of the circle?

2. (20 points.) A homogeneous magnetic field \mathbf{B} is characterized by the vector potential

$$\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}. \quad (4)$$

- (a) Evaluate $\nabla \times \mathbf{A}$. (Hint: $\nabla \times \mathbf{A} = \mathbf{B}$.)
(b) Evaluate $\nabla \cdot \mathbf{A}$.
(c) Is this construction unique? (Hint: Remember the freedom of gauge transformation.)
(d) Now, for the case of $\mathbf{B} = (0, 0, B)$, pointing in the z direction, show that $\mathbf{A} = (0, Bx, 0)$ is a solution. Find another solution.