Homework No. 02 (2019 Spring)

PHYS 420: Electricity and Magnetism II

Due date: Wednesday, 2019 Jan 30, 2:00 PM, in class

1. (**20 points.**) (Based on Griffiths 4th ed. problem 5.45.) In 1897, J. J. Thompson 'discovered' the electron.

   (a) Describe briefly how this discovery influenced the model of an atom.

   (b) Apparently, the experiment involved the measurement of the radius of curvature R of the beam. Suggest a convenient and a reasonably precise method to measure R of a deflected beam. Assume you have the technology available in the times of year 1900. Next, assume you have the technology available in the times of year 2019.

2. (**20 points.**) (Based on Griffiths 4th ed. problem 5.45.)
   A (hypothetical) stationary magnetic monopole q_m held fixed at the origin will have a magnetic field

   $$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{q_m}{r^2}\hat{\mathbf{r}}, \tag{1}$$

   because $\boldsymbol{\nabla}\cdot\mathbf{B} \neq 0$ anymore. Consider the motion of a particle with mass m and electric charge q_e in the field of this magnetic monopole.

   (a) Draw the magnetic field lines of the stationary magnetic monopole.

   (b) Using

   $$\mathbf{F} = q_e\mathbf{v}\times\mathbf{B} \tag{2}$$

   derive the equation of motion for the electric charge to be

   $$\frac{d\mathbf{v}}{dt} = \mathbf{v}\times\mathbf{r}\,\frac{\mu_0}{4\pi}\frac{q_e q_m}{r^3}\frac{1}{m}, \tag{3}$$

   where $\mathbf{v}$ is the velocity of the electric charge q_e.

   (c) Recall that the motion of an electric charge in a uniform magnetic field implies circular (or helical) motion, which in turn implies that the speed $v = |\mathbf{v}|$ is a constant of motion. Show that the speed $v = |\mathbf{v}|$ is a constant of motion even for the motion of an electric charge in the field of a magnetic monopole. That is, show that

   $$\frac{dv}{dt} = 0. \tag{4}$$

   However, the motion is not circular. Nevertheless, it is exactly solvable and the orbit is unbounded and lies on a right circular half-cone with vertex at the monopole. The comments following Eq. (4) are for your information and need not be proved here. Hint: Show that $v^2 = \mathbf{v}\cdot\mathbf{v}$ is a constant of motion. Use $\mathbf{a}\cdot(\mathbf{a}\times\mathbf{b}) = 0$.

3. (**20 points.**) The force $d\mathbf{F}$ on an infinitely small line element $d\mathbf{l}$ of wire, carrying steady current I, placed in a magnetic field $\mathbf{B}$, is

$$d\mathbf{F} = I d\mathbf{l} \times \mathbf{B}. \tag{5}$$

This involves the correspondence

$$q\mathbf{v} \to I d\mathbf{l} \tag{6}$$

for the flow of charge, representing current, in the wire. Consider a wire segment of arbitrary shape (in the shape of a curve C) with one end at the origin and the other end at the tip of vector $\mathbf{L}$. The total force on the segment of wire is given by the line integral

$$\mathbf{F} = \int_{\mathbf{0}\,(\text{path}\,C)}^{\mathbf{L}} I d\mathbf{l} \times \mathbf{B}. \tag{7}$$

Evaluate the total force on a closed loop of wire (of arbitrary shape and carrying steady current I) when it is placed in a uniform magnetic field? Check your result for a loop of wire in the shape of a square in a uniform magnetic field.

4. (**20 points.**) Is it correct to conclude that

$$\boldsymbol{\nabla} \cdot (\mathbf{r} \times \mathbf{A}) = -\mathbf{r} \cdot (\boldsymbol{\nabla} \times \mathbf{A}), \tag{8}$$

where $\mathbf{A}$ is a vector dependent on $\mathbf{r}$? Explain your reasoning.
Context: Magnetic field $\mathbf{B}$ satisfies the Maxwell equation $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$, which implies that a magnetic field can be expressed in terms of a magnetic vector potential $\mathbf{A}$ as $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$.