Homework No. 03 (2019 Spring)

PHYS 420: Electricity and Magnetism II

Due date: Wednesday, 2019 Feb 6, 2:00 PM, in class

1. (**0 points.**) Keywords for finding resource materials: Magnetostatics, Ampere's law, Biot-Savart law, Magnetic dipole-moment. (Chapter 5, Griffiths 4th edition).

2. (**30 points.**) Consider a straight wire of radius a carrying current I described using the current density
$$\mathbf{J}(\mathbf{r}) = \hat{\mathbf{z}}\,\frac{C}{\rho}\,e^{-\lambda\rho}\,\theta(a-\rho), \tag{1}$$
where $\theta(x) = 1$ for $x > 0$ and zero otherwise.
   (a) Find C in terms of the current I.
   (b) Find the magnetic field inside and outside the wire.
   (c) Plot the magnetic field as a function of ρ.

3. (**30 points.**) A steady current I flows down a long cylindrical wire of radius a. The current density in the wire is described by, $n > 0$,
$$\mathbf{J}(\mathbf{r}) = \hat{\mathbf{z}}\,\frac{I}{2\pi a^2}\,(n+2)\left(\frac{\rho}{a}\right)^n \theta(a-\rho). \tag{2}$$
   (a) Show that, indeed,
$$\int_S d\mathbf{S}\cdot\mathbf{J}(\mathbf{r}) = I. \tag{3}$$
   (b) Using Ampere's law show that the magnetic field inside and outside the cylinder is given by
$$\mathbf{B}(\mathbf{r}) = \begin{cases} \dfrac{\mu_0}{4\pi}\dfrac{2I}{\rho}\left(\dfrac{\rho}{a}\right)^{n+2}\hat{\boldsymbol{\phi}} & \rho < a, \\[2ex] \dfrac{\mu_0}{4\pi}\dfrac{2I}{\rho}\,\hat{\boldsymbol{\phi}} & \rho > a. \end{cases} \tag{4}$$
   (c) Plot the magnitude of the magnetic field as a function of ρ.

4. (**20 points.**) The vector potential for a straight wire of infinite extent carrying a steady current I is
$$\mathbf{A}(\mathbf{r}) = \hat{\mathbf{z}}\,\frac{\mu_0 I}{2\pi}\ln\frac{2L}{\rho}, \tag{5}$$
with $L \to \infty$ understood in the equation. The magnetic field around the wire is given by
$$\mathbf{B}(\mathbf{r}) = \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I}{2\pi\rho}. \tag{6}$$

(a) Using an appropriate diagram describe the above vector potential and the magnetic field above.

(b) Evaluate $\nabla \times \mathbf{A}$.

5. (**20 points.**) Magnets are described by their magnetic moment. Estimate the magnetic moment of Earth (assuming it to be a point magnetic dipole $\mathbf{m}$.) Next, similarly, estimate the magnetic moment of a typical refrigerator magnet.