Homework No. 04 (2019 Spring)

PHYS 420: Electricity and Magnetism II

Due date: Monday, 2019 Feb 18, 2:00 PM, in class

0. (**0 points.**) Keywords for finding resource materials: Magnetostatics, Ampere's law, Biot-Savart law, Magnetic vector potential, Magnetic dipole-moment. (Chapter 5, Griffiths 4th edition).

1. (**20 points.**) A steady current I flows through a wire shown in Fig. 1. Find the magnitude and direction of magnetic field at point P. You are given the magnitude of the magnetic

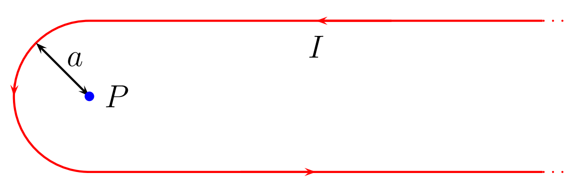

Figure 1: Problem 1.

field due to an infinite length of wire at distance ρ, and a circular loop of wire of radius R at the center of loop, to be

$$B_{\infty\text{-wire}} = \frac{\mu_0 I}{2\pi\rho} \qquad B_{\text{loop}} = \frac{\mu_0 I}{2R}. \tag{1}$$

2. (**20 points.**) Find the magnetic field at the center of a square loop, which carries a steady current I. Let $2L$ be the length of a side, ρ be the distance from center to side, and $R = \sqrt{\rho^2 + L^2}$ be the distance from center to a corner. (Caution: Notation differs from Griffiths.) You should obtain

$$B = \frac{\mu_0 I}{2R} \frac{4}{\pi} \tan\frac{\pi}{4}. \tag{2}$$

Find the magnetic field at the center of a regular pentagon with the same R. Show that the magnetic field at the center of a regular n-sided polygon with same R, carrying a steady current I is

$$B = \frac{\mu_0 I}{2R} \frac{n}{\pi} \tan\frac{\pi}{n}, \tag{3}$$

where R is the distance from center to a corner of the polygon. Show that the magnetic field at the center of a circular loop of radius R,

$$B = \frac{\mu_0 I}{2R}, \tag{4}$$

is obtained in the limit $n \to \infty$.

3. (**20 points.**) The vector potential for a point magnetic dipole moment $\mathbf{m}$ is given by

$$\mathbf{A} = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \mathbf{r}}{r^3}. \tag{5}$$

Determine the corresponding magnetic field due to the point dipole using

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}. \tag{6}$$

Find the simplified expression for the magnetic field everywhere along the line collinear to the magnetic moment $\mathbf{m}$. Next, find the simplified expression for the magnetic field in the plane containing the magnetic moment and perpendicular to the magnetic moment $\mathbf{m}$.

4. (**20 points.**) The electric force (in the regime of electrostatics) is a conservative force. It allows us to define a electric potential energy. Is magnetic force a conservative force? What is the energy associated with a particle moving in a magnetic field. Note that quantum mechanics is formulated in terms of energy.

5. (**20 points.**) Use online resource to find the expression for the perimeter of an ellipse.