

Homework No. 06 (2019 Spring)

PHYS 420: Electricity and Magnetism II

Due date: Wednesday, 2019 Mar 6, 2:00 PM, in class

0. (**0 points.**) Keywords for finding resource materials: Electromagnetic energy conservation; Electromagnetic momentum conservation; Monochromatic electromagnetic wave.
1. (**60 points.**) A plane wave is incident, in vacuum, on a perfectly absorbing flat screen.
 - (a) Without compromising generality we can choose the screen at $z = z_a$. Starting with the statement of conservation of linear momentum,

$$\frac{\partial \mathbf{G}}{\partial t} + \nabla \cdot \mathbf{T} + \mathbf{f} = 0, \quad (1)$$

integrate on the volume between $z = z_a - \delta$ and $z = z_a + \delta$ for infinitely small $\delta > 0$. Interpret the integral of force density \mathbf{f} as the total force, \mathbf{F} , on the plate. Further, note that the integral of momentum density \mathbf{G} goes to zero for infinitely small δ . Thus, obtain

$$\mathbf{F} = - \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{z_a - \delta}^{z_a + \delta} dz \nabla \cdot \mathbf{T}. \quad (2)$$

- (b) Use divergence theorem to conclude

$$\mathbf{F} = - \oint d\mathbf{a} \cdot \mathbf{T}, \quad (3)$$

where the closed surface encloses the volume between $z = z_a - \delta$ and $z = z_a + \delta$ for infinitely small $\delta > 0$. Choose the plane wave to be incident on the side $z = z - \delta$ of the plate, and assuming $\mathbf{E} = 0$ and $\mathbf{B} = 0$ on the side $z = z + \delta$, conclude that

$$\frac{\mathbf{F}}{A} = \hat{\mathbf{z}} \cdot \mathbf{T}|_{z=z_a - \delta}, \quad (4)$$

where A is the total area of the screen. The electromagnetic stress tensor \mathbf{T} in these expressions is given by

$$\mathbf{T} = \mathbf{1}U - (\mathbf{D}\mathbf{E} + \mathbf{B}\mathbf{H}), \quad (5)$$

where U is the electromagnetic energy density,

$$U = \frac{1}{2}(\mathbf{D} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{H}). \quad (6)$$

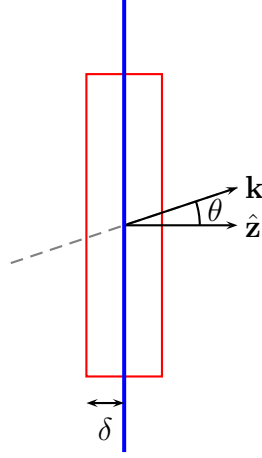


Figure 1: A plane wave with direction of propagation \mathbf{k} incident on a screen.

- (c) For the particular case when the plane wave is incident normally on the screen ($\theta = 0$ in Fig. 1) calculate the force per unit area in the direction normal to the screen by evaluating

$$\frac{\mathbf{F} \cdot \hat{\mathbf{z}}}{A}. \quad (7)$$

Express the answer in terms of U using the properties of a plane wave: $\mathbf{k} \cdot \mathbf{E} = 0$, $\mathbf{k} \cdot \mathbf{B} = 0$, $\mathbf{E} \cdot \mathbf{B} = 0$, $|\mathbf{E}| = c|\mathbf{B}|$, and $kc = \omega$.

- (d) Consider the case when the plane wave is incident obliquely on the screen such that $\hat{\mathbf{k}} \cdot \hat{\mathbf{z}} = \cos \theta$ and $\mathbf{H} \cdot \hat{\mathbf{z}} = 0$. Calculate the force per unit area in the direction normal to the screen by evaluating

$$\frac{\mathbf{F} \cdot \hat{\mathbf{z}}}{A}, \quad (8)$$

and the force per unit area tangential to the screen by evaluating

$$\frac{\mathbf{F} \cdot \hat{\mathbf{x}}}{A}. \quad (9)$$

Express the answer in terms of U and θ using the properties of a plane wave.