Homework No. 08 (2019 Spring)

PHYS 420: Electricity and Magnetism II

Due date: Wednesday, 2019 Mar 27, 2:00 PM, in class

0. (**0 points.**) Keywords for finding resource materials: Special relativity, Lorentz transformation, time dilation, length contraction, relativistic velocity addition formula.

1. (**20 points.**) The Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \qquad \beta = \frac{v}{c}. \tag{1}$$

   (a) Evaluate γ for $v = 30\,\mathrm{m/s}$ ($\sim 70\,\mathrm{miles/hour}$).

   (b) Evaluate γ for $v = 3c/5$.

2. (**20 points.**) Lorentz transformation describing a boost in the x-direction is obtained using the matrix

$$L = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{2}$$

   (a) Show that the determinant of the matrix L is 1.

   (b) Determine L^{-1}.

3. (**60 points.**) The Poincaré formula for the addition of (parallel) velocities is

$$v = \frac{v_a + v_b}{1 + \dfrac{v_a v_b}{c^2}}, \tag{3}$$

   where v_a and v_b are velocities and c is speed of light in vacuum. Jerzy Kocik, from the department of Mathematics in SIUC, has invented a geometric diagram that allows one to visualize the Poincaré formula. (Refer [1].) An interactive applet for exploring velocity addition is available at Kocik's web page [2]. (For the following assume that the Poincaré formula holds for all speeds, subluminal ($v_i < c$), superluminal ($v_i > c$), and speed of light.)

   (a) Analyse what is obtained if you add two subluminal speeds?

   (b) Analyse what is obtained if you add a subluminal speed to speed of light?

   (c) Analyse what is obtained if you add a subluminal speed to a superluminal speed?

   (d) Analyse what is obtained if you add speed of light to another speed of light?

   (e) Analyse what is obtained if you add a superluminal speed to speed of light?

   (f) Analyse what is obtained if you add two superluminal speeds?

4. (**30 points.**) Let

$$\tanh\theta = \beta, \tag{4}$$

   where $\beta = v/c$. Addition of (parallel) velocities in terms of the parameter θ obeys the arithmatic addition

$$\theta = \theta_a + \theta_b. \tag{5}$$

(a) Invert the expression in Eq. (4) to find the explicit form of θ in terms of β as a logarithm.

(b) Show that Eq. (5) leads to the relation

$$\left(\frac{1+\beta}{1-\beta}\right) = \left(\frac{1+\beta_a}{1-\beta_a}\right)\left(\frac{1+\beta_b}{1-\beta_b}\right). \tag{6}$$

(c) Using Eq. (6) derive the Poincaré formula for the addition of (parallel) velocities.

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[1] J. Kocik, "Geometric diagram for relativistic addition of velocities," Am. J. Phys. **80**, 737–739 (2012), arXiv:1408.2435.

[2] J. Kocik, "An interactive applet for exploring relativistic velocity addition," http://lagrange.math.siu.edu/Kocik/relativity/diagram.html.