Homework No. 10 (2019 Spring)

PHYS 420: Electricity and Magnetism II

Due date: Monday, 2019 Apr 8, 2:00 PM, in class

0. (**0 points.**) Keywords for finding resource materials: Relativistic force; Motion under uniform proper acceleration; Relativistic hyperbolic motion.

1. (**20 points.**) The path of a relativistic particle moving along a straight line with constant (proper) acceleration α is described by the equation of a hyperbola

$$z^2 - c^2t^2 = z_0^2, \qquad z_0 = \frac{c^2}{\alpha}. \tag{1}$$

This is the motion of a particle 'dropped' from $z = z_0$ at $t = 0$ in region of constant (proper) acceleration. See Figure 1. Using geometric (diagrammatic) arguments might be easiest to answer the following.

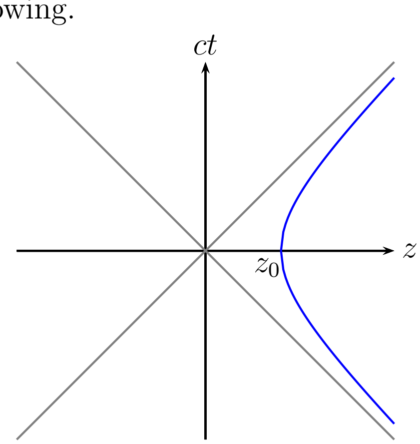

Figure 1: Problem 1

(a) Will a photon dispatched to 'chase' this particle at $t = 0$ from $z = 0$ ever catch up with it? If yes, when and where does it catch up?

(b) Will a photon dispatched to 'chase' this particle at $t = 0$ from $0 < z < z_0$ ever catch up with it? If yes, when and where does it catch up?

(c) Will a photon dispatched to 'chase' this particle, at $t = 0$ from $z < 0$ ever catch up with it? If yes, when and where does it catch up?

What are the implications for the observable part of our universe from this analysis?

2. (**20 points.**) The path of a relativistic particle 1 moving along a straight line with constant (proper) acceleration g is described by the equation of a hyperbola

$$z_1(t) = \sqrt{c^2t^2 + z_0^2}, \qquad z_0 = \frac{c^2}{g}. \tag{2}$$

This is the motion of a particle that comes to existance at $z_1 = +\infty$ at $t = -\infty$, then 'falls' with constant (proper) acceleration g. If we choose $x_q(0) = 0$ and $y_q(0) = 0$, the particle 'falls' keeping itself on the z-axis, comes to stop at $z = z_0$, and then returns back to infinity. Consider another relavistic particle 2 undergoing hyperbolic motion given by

$$z_2(t) = -\sqrt{c^2t^2 + z_0^2}, \qquad z_0 = \frac{c^2}{g}. \tag{3}$$

This is the motion of a particle that comes to existance at $z_2 = -\infty$ at $t = -\infty$, then 'falls' with constant (proper) acceleration g. If we choose $x_q(0) = 0$ and $y_q(0) = 0$, the particle 'falls' keeping itself on the z-axis, comes to stop at $z = -z_0$, and then returns back to negative infinity. The world-line of particle 1 is the blue curve in Figure 2, and the world-line of particle 2 is the red curve in Figure 2. Imagine the particles are sources of light (imagine a flash light pointing towards origin).

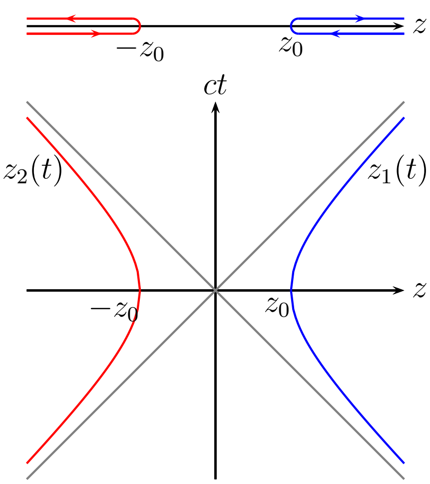

Figure 2: Problem 2

(a) At what time will the light from particle 1 first reach particle 2? Where are the particles when this happens?

(b) At what time will the light from particle 2 first reach particle 1? Where are the particles when this happens?

(c) Can the particles communicate with each other?

(d) Can the particles ever detect the presence of the other? In other words, can one particle be aware of the existence of the other? What can you deduce about our observable universe from this?