Homework No. 12 (2019 Spring)

PHYS 420: Electricity and Magnetism II

Due date: Wednesday, 2019 May 1, 2:00 PM, in class

0. (**0 points.**) Keywords for finding resource materials: Retarded time; Radiation; Larmor formula.

1. (**20 points.**) An electron of charge e and mass m moves in a nearly circular orbit under the Coulomb forces produced by a proton. Suppose, as it radiates, the electron continues to move on a circle of ever decreasing radii.

   (a) The equation of motion for the electron given by Newton's laws of motion is

   $$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}, \tag{1}$$

   where the acceleration of the election is the centripetal acceleration

   $$a = \frac{v^2}{r}. \tag{2}$$

   The total energy of the system E is the sum of the kinetic energy and electrostatic potential energy. Show that

   $$E = \frac{1}{2}mv^2 - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = -\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}. \tag{3}$$

   (b) A charge that is accelerating will loose energy in the form of radiation. The Larmor formula

   $$P = -\frac{dE}{dt} = \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{3c^3}a^2, \tag{4}$$

   gives the rate of loss of energy, the power P.

   (c) Combine the equation of motion of the electron with the Larmor formula to construct the following differential equation for the radius r,

   $$\frac{1}{c}\frac{dr}{dt} = -\frac{4}{3}\frac{r_0^2}{r^2}, \tag{5}$$

   where $r_0 \sim 3\times 10^{-15}$ m is the classical radius of the electron defined using the equality

   $$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_0} = mc^2. \tag{6}$$

Solve this differential equation. In a finite time the electron reaches the center. Calculate how long it takes for the electron to hit the proton if it starts from an initial radius $a_0 \sim 0.5 \times 10^{-10}$ m, the Bohr radius. This is the classical lifetime of a Bohr atom.

The following article by J. D. Olsen and K. T. McDonald titled 'Classical Lifetime of a Bohr Atom' available at

http://www.physics.princeton.edu/~mcdonald/examples/orbitdecay.pdf

is recommended for reading.

(d) Most atoms have lifetimes greater than the age of the universe, which is about 10^{17} s. This instability was one of the reasons for the discovery of quantum mechanics.