Midterm Exam No. 01 (Spring 2019)

PHYS 510: Classical Mechanics

Date: 2019 Feb 14

1. (**20 points.**) Given the functional

$$F[u] = \int_{x_1}^{x_2} dx\, x\, u(x) \left(\frac{du}{dx}\right)^2. \tag{1}$$

Assuming no variations at the end points, evaluate the functional derivative

$$\frac{\delta F[u]}{\delta u(x)}. \tag{2}$$

2. (**20 points.**) Find the geodesics on the surface of a right circular cylinder. Identify the curves.

3. (**20 points.**) The motion of a particle of mass m undergoing simple harmonic motion is described by

$$\frac{d}{dt}(mv) = -kx, \tag{3}$$

where $v = dx/dt$ is the velocity in the x direction.

   (a) Find the Lagrangian for this system that implies the equation of motion of Eq. (3).

   (b) Determine the canonical momentum p for this system.

   (c) Determine the Hamilton $H(p, x)$ for this system.

4. (**20 points.**) A relativistic particle of mass m is described by the Lagrangian

$$L(\mathbf{x}, \mathbf{v}) = -mc^2\sqrt{1 - \frac{v^2}{c^2}}, \tag{4}$$

where $\mathbf{x}$ is the position of the particle and and $\mathbf{v}$ is the velocity of the particle.

   (a) Determine the momentum of $\mathbf{p}$ the relativistic particle.

   (b) Determine the relativistic equation of motion.

   (c) Determine the Hamilton $H(\mathbf{p}, \mathbf{r})$ for the particle.