Homework No. 01 (Spring 2019)

PHYS 510: Classical Mechanics

Due date: Thursday, 2019 Jan 24, 4.30pm

1. (**20 points.**) Given
$$F[u] = \int_{x_1}^{x_2} dx\, a(x) \frac{du(x)}{dx}. \tag{1}$$
Evaluate
$$\frac{\delta F}{\delta u(x)}. \tag{2}$$

2. (**20 points.**) (Gelfand and Fomin, Calculus of Variations.) Evaluate the functional derivatives of the following functionals, assuming no variation at the end points.

   (a)
$$F[y] = \int_0^1 dx\, \frac{dy}{dx} \tag{3}$$

   (b)
$$F[y] = \int_0^1 dx\, y \frac{dy}{dx} \tag{4}$$

   (c)
$$F[y] = \int_0^1 dx\, xy \frac{dy}{dx} \tag{5}$$

   (d)
$$F[y] = \int_a^b \frac{dx}{x^3} \left(\frac{dy}{dx}\right)^2 \tag{6}$$

3. (**20 points.**) The distance between two points in a plane is characterized by the infinitesimal statement
$$ds^2 = dx^2 + dy^2. \tag{7}$$
Let us prove the intuitively obvious statement that the curve of shortest distance going through two points (x_1, y_1) and (x_2, y_2) in a plane, the geodesics of a plane, is a straight line passing through the two points. This amounts to finding the extremal of the functional
$$l[y] = \int_{(x_1,y_1)}^{(x_2,y_2)} ds = \int_{(x_1,y_1)}^{(x_2,y_2)} dx \sqrt{1 + \left(\frac{dy}{dx}\right)^2}. \tag{8}$$

Using the fact that we require the curve to necessarily pass through the points (x_1, y_1) and (x_2, y_2), show that

$$\frac{\delta l[y]}{\delta y(x)} = -\frac{d}{dx}\left[\frac{\frac{dy}{dx}}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}\right]. \tag{9}$$

Then, using the condition that the functional derivative is zero for the extremal curve, derive the equation of the geodesic to be given by

$$\frac{dy}{dx} = c, \tag{10}$$

where c is a contant. Identify this as the equation of a straight line in a plane. Find c.