Homework No. 05 (Spring 2019)

PHYS 520B: Electromagnetic Theory

Due date: Tuesday, 2019 Apr 2, 12.35pm

1. (**20 points.**) Evaluate the integral

$$\zeta(s) = \lim_{\epsilon\to 0+} \int_\epsilon^\infty dx \left(\frac{\pi}{x}\right)^s \delta(\sin x) \tag{1}$$

as a sum. The resultant sum is the Riemann zeta function. Determine $\zeta(2)$.
Hint: Use the identity

$$\delta(F(x)) = \sum_r \frac{\delta(x-a_r)}{\left|\frac{dF}{dx}\Big|_{x=a_r}\right|}, \tag{2}$$

where the sum on r runs over the roots a_r of the equation $F(x) = 0$.

2. (**20 points.**) A charged particle with charge q moves on the z-axis with constant speed v, $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$. The scalar and vector potential generated by this charged particle is

$$\phi(\mathbf{r},t) = \gamma\frac{q}{4\pi\varepsilon_0}\frac{1}{\sqrt{(x^2+y^2)+\gamma^2(z-vt)^2}}, \tag{3a}$$

$$c\mathbf{A}(\mathbf{r},t) = \beta\gamma\frac{q}{4\pi\varepsilon_0}\frac{\hat{\mathbf{z}}}{\sqrt{(x^2+y^2)+\gamma^2(z-vt)^2}}. \tag{3b}$$

(a) Using

$$\mathbf{E} = -\boldsymbol{\nabla}\phi - \frac{\partial}{\partial t}\mathbf{A}, \tag{4a}$$

$$\mathbf{A} = \boldsymbol{\nabla}\times\mathbf{A}, \tag{4b}$$

evaluate the electric and magnetic field generated by the charged particle to be

$$\mathbf{E}(\mathbf{r},t) = \gamma\frac{q}{4\pi\varepsilon_0}\frac{x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+(z-vt)\hat{\mathbf{k}}}{[(x^2+y^2)+\gamma^2(z-vt)^2]^{\frac{3}{2}}}, \tag{5a}$$

$$c\mathbf{B}(\mathbf{r},t) = \beta\gamma\frac{q}{4\pi\varepsilon_0}\frac{-y\hat{\mathbf{i}}+x\hat{\mathbf{j}}}{[(x^2+y^2)+\gamma^2(z-vt)^2]^{\frac{3}{2}}}. \tag{5b}$$

(b) Evaluate the electromagnetic momentum density for this configuration by evaluating

$$\mathbf{G}(\mathbf{r},t) = \varepsilon_0\mathbf{E}(\mathbf{r},t)\times\mathbf{B}(\mathbf{r},t). \tag{6}$$

3. (**20 points.**) The following problem is a challenge problem. Find the fields for a charged particle with charge q undergoing hyperbolic motion while moving on the z-axis, described by

$$\mathbf{r}_q(t) = \hat{\mathbf{x}}\,0 + \hat{\mathbf{y}}\,0 + \hat{\mathbf{z}}\sqrt{c^2t^2 + z_0^2}. \tag{7}$$

Study the article by Franklin and Griffiths (arXiv:1405.7729). Try to reproduce the results there as much as you can. Do a forward literature search, that is, find the articles referring back to this article. Summarize the latest status of this conundrum.