Homework No. 07 (Spring 2019)

PHYS 520B: Electromagnetic Theory

Due date: Tuesday, 2019 Apr 23, 12.35pm

1. (**20 points.**) A particle, of charge q and mass m, always moves with speed $v \ll c$.

   (a) Consider the case when it oscillates on the x-axis with frequency ω_0 and amplitude A given by

   $$\mathbf{r}_1(t) = \hat{\mathbf{x}} A \cos \omega_0 t. \tag{1}$$

   Obtain expressions for the radiated electric field $\mathbf{E}(\mathbf{r}, t)$, radiated magnetic field $\mathbf{B}(\mathbf{r}, t)$, angular distribution of the radiated power $dP/d\Omega$, and the total power radiated P.

   (b) Next, consider the case when the particle moves on a circle described by

   $$\mathbf{r}_2(t) = \hat{\mathbf{x}} A \cos \omega_0 t + \hat{\mathbf{y}} A \sin \omega_0 t. \tag{2}$$

   Obtain expressions for the radiated electric field $\mathbf{E}(\mathbf{r}, t)$, radiated magnetic field $\mathbf{B}(\mathbf{r}, t)$, angular distribution of the radiated power $dP/d\Omega$, and the total power radiated P.

   (c) Show that the radiated electric and magnetic field is additive, that is, it is the sum of two oscillators.

   (d) Show that the radiated power is not additive, but exhibits interference effects. Identify the interference term for the circular motion.

   (e) Find directions $\hat{\mathbf{r}}$ for which the interference term goes to zero.

2. (**20 points.**) An electron of charge e and mass m moves in a circular orbit under the Coulomb forces produced by a proton. Suppose, as it radiates, the electron continues to move on a circle.

   (a) Determine the acceleration a of the electron using Newton's laws of motion.

   (b) Show that the energy of the system is given by

   $$E = \frac{1}{2} m v^2 - \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r} = -\frac{1}{2} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r}. \tag{3}$$

   (c) Using the Larmor formula

   $$P = -\frac{dE}{dt} = \frac{1}{4\pi\varepsilon_0} \frac{2e^2}{3c^3} a^2, \tag{4}$$

   construct a differential equation for E.

(d) Show that

$$\frac{dE}{dt} = \frac{1}{2}ma\frac{dr}{dt}. \tag{5}$$

Thus, construct a differential equation for r.

(e) In a finite time the electron reaches the center. Calculate how long it takes for the electron to hit the proton if it starts from an initial radius of $r_{\text{initial}} = 10^{-10}\,\text{m}$.

(This instability was one of the reasons for the discovery of quantum mechanics.)