Midterm Exam No. 01 (Fall 2019)

PHYS 301: Theoretical Methods in Physics

Date: 2019 Sep 18

Note: Standard identities will be provided to a student when requested.

1. (**20 points.**) Find the real and imaginary part of the function

$$f = \cos z. \tag{1}$$

   Here z represents a complex number.

2. (**20 points.**) Find the three roots of -1 by solving the equation

$$z^3 = -1. \tag{2}$$

   Mark the the points corresponding to the three roots on the complex plane.

3. (**20 points.**) Check if the function

$$f(z) = e^{z+iz} \tag{3}$$

   satisfies the Cauchy-Riemann conditions.

4. (**20 points.**) Evalauate the contour integral

$$I(a) = \frac{1}{2\pi i} \oint_c dz \frac{\sin z}{(z^2 + a^2)}, \tag{4}$$

   where the contour c is a unit circle going counterclockwise with center at the origin. Let a be a real number. Express your answer in simplified form.

5. (**20 points.**) Find the eigenvalues and eigenvectors of the matrix

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{5}$$