Midterm Exam No. 02 (Fall 2019)

PHYS 301: Theoretical Methods in Physics

Date: 2019 Oct 16

Note: Standard identities will be provided to a student when requested.

1. (**20 points.**) Using the property of Kronecker δ-function and Levi-Civita symbol in three dimensions evaluate the following using index notation.

$$\delta_{ij}\delta_{ji} = \tag{1a}$$

$$\delta_{ij}\varepsilon_{ijk} = \tag{1b}$$

2. (**20 points.**) Evaluate the left hand side of the equation

$$\boldsymbol{\nabla}\frac{1}{r^3} = \alpha\,\hat{\mathbf{r}}\,r^n. \tag{2}$$

Thus find α and n.

3. (**10 points.**) Evaluate the left hand side of the equation

$$\boldsymbol{\nabla}(\mathbf{r}\cdot\mathbf{p}) = a\,\mathbf{p} + b\,\mathbf{r}, \tag{3}$$

where $\mathbf{p}$ is a constant vector. Thus find a and b.

4. (**20 points.**) In terms of the unit vectors

$$\hat{\boldsymbol{\rho}} = \cos\phi\,\hat{\mathbf{i}} + \sin\phi\,\hat{\mathbf{j}} + 0\,\hat{\mathbf{k}}, \tag{4a}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{i}} + \cos\phi\,\hat{\mathbf{j}} + 0\,\hat{\mathbf{k}}, \tag{4b}$$

$$\hat{\mathbf{z}} = 0\,\hat{\mathbf{i}} + 0\,\hat{\mathbf{j}} + \hat{\mathbf{k}}, \tag{4c}$$

where $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are basis vectors in rectangular coordinate system the basis vectors in cylindrical polar coordinates are

$$\mathbf{e}_1 = \hat{\boldsymbol{\rho}}, \qquad \mathbf{e}_2 = \rho\hat{\boldsymbol{\phi}}, \qquad \mathbf{e}_3 = \hat{\mathbf{z}}. \tag{5}$$

Evaluate all the components of the metric tensor

$$g_{ij} = \mathbf{e}_i\cdot\mathbf{e}_j. \tag{6}$$

5. (**20 points.**) The eigenvectors for the Stern-Gerlach Hamiltonian, choosing $\phi = 0$, are

$$|+;\theta\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{pmatrix} \quad \text{and} \quad |-;\theta\rangle = \begin{pmatrix} -\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix}. \tag{7}$$

Also, we have

$$|+;\theta\rangle\langle+;\theta| = \frac{1}{2}\begin{pmatrix} 1+\cos\theta & \sin\theta \\ \sin\theta & 1-\cos\theta \end{pmatrix} \quad \text{and} \quad |-;\theta\rangle\langle-;\theta| = \frac{1}{2}\begin{pmatrix} 1-\cos\theta & -\sin\theta \\ -\sin\theta & 1+\cos\theta \end{pmatrix}. \tag{8}$$

Using the notation for the probability for a measurement in the Stern-Gerlach experiment, introduced in the class, evaluate

$$p\big([+;0] \to [-;\frac{\pi}{3}] \to [+;0]\big). \tag{9}$$