Midterm Exam No. 03 (Fall 2019)

PHYS 301: Theoretical Methods in Physics

Date: 2019 Nov 18

Note: Standard identities will be provided to a student when requested.

1. (**20 points.**) Fourier series (or transformation) is defined as ($0 \leq \phi < 2\pi$)

$$f(\phi) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} e^{im\phi} a_m, \tag{1}$$

where the coefficients a_m are determined using

$$a_m = \int_0^{2\pi} d\phi \, e^{-im\phi} f(\phi). \tag{2}$$

Determine all the Fourier components a_m for the function $(1+\cos^2 \phi)$.

2. (**20 points.**) The half-range Fourier space is spanned by the Fourier eigenfunctions

$$\sin m\phi, \qquad m = 1, 2, 3, \ldots, \qquad 0 \leq \phi \leq \pi. \tag{3}$$

An arbitrary function $f(\phi)$, for ϕ limited to half the range, has the half-range Fourier series representation

$$f(\phi) = \sum_{m=1}^{\infty} a_m \sin m\phi, \tag{4}$$

where $\sin m\phi$ are the half-range Fourier eigenfunctions and a_m are the respective half-range Fourier components given by

$$a_m = \frac{2}{\pi} \int_0^{\pi} d\phi \, \sin m\phi \, f(\phi). \tag{5}$$

Determine all the half-range Fourier components a_m for the function

$$f(\phi) = \sin 2\phi. \tag{6}$$

3. (**20 points.**) Evaluate the integral

$$\int_{-1}^{1} dx \, \delta(1 - 2x) \Big[8x^2 + 2x - 1 \Big]. \tag{7}$$

(Caution: Be careful to avoid a possible error in sign.)

4. (**20 points.**) Evaluate the integral

$$\int_{-\infty}^{\infty} dx\, \delta(x) \frac{1}{\sqrt{x^2 + a^2}}. \tag{8}$$

5. (**20 points.**) Consider the differential equation satisfied by $\phi(x)$,

$$\frac{d^2\phi}{dx^2} = k^2 \phi, \tag{9}$$

with constraints

$$\phi(0) = 0, \tag{10a}$$

$$\left.\frac{d\phi}{dx}\right|_{x=0} = L. \tag{10b}$$

Find the solution for $\phi(x)$ in terms of k and L.