Final Exam (Fall 2019)

PHYS 320: Electricity and Magnetism I

Date: 2019 Dec 9

Note: Standard identities will be provided to a student when requested.

1. (**20 points.**) The electric potential due to a point dipole $\mathbf{p}$ is given by

$$\phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\mathbf{r}}{r^3}. \tag{1}$$

   Evaluate the corresponding electric field using

$$\mathbf{E} = -\boldsymbol{\nabla}\phi. \tag{2}$$

2. (**20 points.**) Fourier series (or transformation) is defined as ($0 \le \phi < 2\pi$)

$$f(\phi) = \frac{1}{2\pi}\sum_{m=-\infty}^{\infty} e^{im\phi}a_m, \tag{3}$$

   where the coefficients a_m are determined using

$$a_m = \int_0^{2\pi} d\phi\, e^{-im\phi} f(\phi). \tag{4}$$

   Determine all the Fourier components a_m for the function $\sin^2\phi$.

3. (**20 points.**) The Legendre polynomials of order l, $-1 \le x \le 1$, are

$$P_l(x) = \left(\frac{d}{dx}\right)^l \frac{(x^2-1)^l}{2^l l!}. \tag{5}$$

   In particular,

$$P_0(x) = 1, \tag{6a}$$
$$P_1(x) = x, \tag{6b}$$
$$P_2(x) = \frac{3}{2}x^2 - \frac{1}{2}. \tag{6c}$$

   Express the function

$$\sigma(\theta) = \cos^2\theta \tag{7}$$

   in terms of Legendre polynomials.

4. (**20 points.**) Two charges each with charge $+q$ is placed at positions $\mathbf{r}_1 = a\,\hat{\mathbf{i}}$ and $\mathbf{r}_2 = -a\,\hat{\mathbf{i}}$. A third charge with charge $-2q$ is placed at the origin. Find the quadruple moment of this configuration of three charges.

5. (**20 points.**) Consider a uniformly polarized half-slab, that occupies half of space, and has the direction of its polarization in the direction $\hat{\mathbf{z}}$ normal to the surface of slab, described by

$$\mathbf{P}(\mathbf{r}) = \sigma\,\hat{\mathbf{z}}\,\theta(-z), \tag{8}$$

where σ is the polarization per unit area of the slab. Determine the effective charge density by evaluating

$$\rho_{\text{eff}}(\mathbf{r}) = -\boldsymbol{\nabla}\cdot\mathbf{P}. \tag{9}$$

Draw a diagram illustrating how the distribution of dipole moment density $\mathbf{P}$ leads to a surface charge density.

6. (**20 points.**) A grounded perfectly conducting thin plate is located at $z = 0$ plane. A positive charge q is placed at $\mathbf{r}_1 = a\,\hat{\mathbf{z}}$. A negative charge $-q$ is placed at $\mathbf{r}_2 = 2a\,\hat{\mathbf{z}}$.

   (a) Determine the magnitude and direction of the electrostatic force on the positive charge due to the negative charge.

   (b) Determine the magnitude and direction of the electrostatic force on the positive charge due to the plate. Use method of images.

   (c) Determine the magnitude and direction of the total electrostatic force on the positive charge.