Midterm Exam No. 01 (Fall 2019)

PHYS 320: Electricity and Magnetism I

Date: 2019 Sep 13

1. (**20 points.**) Using the property of Kronecker δ-function and Levi-Civita symbol in three dimensions evaluate the following using index notation.

$$\delta_{ij}\delta_{ij} = \tag{1a}$$

$$\varepsilon_{ijk}\delta_{jm}\delta_{ki} = \tag{1b}$$

2. (**20 points.**) Given

$$\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r} \tag{2}$$

where $\mathbf{B}$ is a constant (homogeneous in space) vector field. Determine the numbers α and β in

$$\boldsymbol{\nabla} \times \mathbf{A} = \alpha\mathbf{B} + \beta\mathbf{r} \tag{3}$$

that would make the equation an identity. Here the vector $\mathbf{r}$ represents a coordinate in space and $\boldsymbol{\nabla}$ is the gradient operator.

3. (**20 points.**) Evaluate the left hand side of the equation

$$\boldsymbol{\nabla} r^3 = \alpha\, \hat{\mathbf{r}}\, r^n. \tag{4}$$

Thus find α and n.

4. (**20 points.**) Evaluate the integral

$$\int_0^1 dx\, \delta(1-2x)\Big[2x-1\Big]. \tag{5}$$

5. (**20 points.**) Evaluate the vector area of a spherical ball of radius R using

$$\mathbf{a} = \int_S d\mathbf{a}, \tag{6}$$

where S stands for the surface of the spherical ball. (Caution: The question is discussing the vector area, which is different from the typical surface area of a sphere.)

6. (**20 points.**) A thick spherical shell carries charge density

$$\rho(\mathbf{r}) = \begin{cases} 0, & r < a, \\ \dfrac{1}{4\pi r^2}\dfrac{Q}{(b-a)}, & a \le r \le b, \\ 0, & b < r. \end{cases} \tag{7}$$

Find the electric field in the three regions: (i) $r < a$, (ii) $a < r < b$, (iii) $b < r$. Plot the magnitude of the electric field as a function of r, for the case $b = 2a$.