Midterm Exam No. 02 (Fall 2019)

PHYS 320: Electricity and Magnetism I

Date: 2019 Oct 11

Note: Standard identities will be provided to a student when requested.

1. (**20 points.**) Three protons are placed at the corners of an equilateral triangle of side a.
   (a) Determine the electric potential at the center of the triangle.
   (b) How much potential energy is required to move another proton from infinity to the center of the triangle?

2. (**20 points.**) The electric potential for a charge distribution is exactly described by

$$\phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{c}\cdot\mathbf{r}}{r^3}, \qquad \mathbf{r} \neq 0, \tag{1}$$

   where $\mathbf{c}$ is a property of the charge distribution.
   (a) Evaluate the corresponding electric field using

$$\mathbf{E} = -\boldsymbol{\nabla}\phi. \tag{2}$$

   (b) Draw the electric field lines. Further, draw a picture illustrating the features of the charge distribution described by $\mathbf{c}$.

3. (**20 points.**) The electric field due to a point dipole $\mathbf{d}$ at a distance $\mathbf{r}$ away from dipole is given by the expression

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\big[3(\mathbf{d}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{d}\big]. \tag{3}$$

   Consider the case when the point dipole is positioned at the origin and is pointing in the z-direction, i.e., $\mathbf{d} = d\,\hat{\mathbf{z}}$.
   (a) Qualitatively draw the electric field lines for the dipole $\mathbf{d}$.
   (b) Find the (simplified) expression for the electric field everywhere on the x-axis. (Hint: On the positive x-axis we have, $\hat{\mathbf{r}} = \hat{\mathbf{x}}$ and $r = x$.) Plot the magnitude of the electric field on the x-axis as a function of x.

4. (**20 points.**) For what a, b, and $\mathbf{c}$, is the relation

$$\boldsymbol{\nabla}\left[\frac{(\mathbf{d}_1\cdot\hat{\mathbf{r}})(\mathbf{d}_2\cdot\hat{\mathbf{r}})}{r^3}\right] = \frac{a\,(\mathbf{d}_1\cdot\hat{\mathbf{r}})\,\mathbf{d}_2 + b\,(\mathbf{d}_2\cdot\hat{\mathbf{r}})\,\mathbf{d}_1 + (\mathbf{d}_1\cdot\hat{\mathbf{r}})(\mathbf{d}_2\cdot\hat{\mathbf{r}})\,\mathbf{c}}{r^4} \tag{4}$$

   an identity. What are the dimensions of a, b, and $\mathbf{c}$?