Midterm Exam No. 03 (Fall 2019)

PHYS 320: Electricity and Magnetism I

Date: 2019 Nov 8

Note: Standard identities will be provided to a student when requested.

1. (**20 points.**) Fourier series (or transformation) is defined as ($0 \leq \phi < 2\pi$)

$$f(\phi) = \frac{1}{2\pi} \sum_{-\infty}^{\infty} e^{im\phi} a_m, \tag{1}$$

where the coefficients a_m are determined using

$$a_m = \int_0^{2\pi} d\phi \, e^{-im\phi} f(\phi). \tag{2}$$

Determine all the Fourier components a_m for the function $(1+\cos^2\phi)$.

2. (**20 points.**) The Legendre polynomials $P_l(x)$ of order $l = 0, 1, 2, \ldots$ satisfy the recurrence relation

$$(2l+1)xP_l(x) = (l+1)P_{l+1}(x) + lP_{l-1}(x), \qquad l = 1, 2, 3, \ldots. \tag{3}$$

Recall,

$$P_0(x) = 1, \tag{4a}$$
$$P_1(x) = x. \tag{4b}$$

Derive the explicit expression for $P_3(x)$ using the recurrence relation.

3. (**20 points.**) Consider the electric potential due to a solid sphere with uniform charge density Q. The angular integral in this evaluation involves the integral

$$\frac{1}{2} \int_{-1}^{1} dt \frac{1}{\sqrt{r^2 + r'^2 - 2rr't}}. \tag{5}$$

Evaluate the integral for $r < r'$ and $r' < r$, where r and r' are distances measured from the center of the sphere. (Hint: Substitute $r^2 + r'^2 - 2rr't = y$.)

4. (**20 points.**) Two charges each with charge $+q$ is placed at positions $\mathbf{r}_1 = a\,\hat{\mathbf{i}}$ and $\mathbf{r}_2 = a\,\hat{\mathbf{j}}$. A third charge with charge $-2q$ is placed at the origin. Find the monopole moment and the dipole moment of this configuration of three charges.

5. (**20 points.**) Two charges, each with charge $+q$, are placed at positions $\mathbf{r}_1 = a\,\hat{\mathbf{i}}$ and $\mathbf{r}_2 = -a\,\hat{\mathbf{i}}$. Another set of two charges, each with charge $-q$, are placed at positions $\mathbf{r}_3 = a\,\hat{\mathbf{j}}$ and $\mathbf{r}_4 = -a\,\hat{\mathbf{j}}$. Find the monopole moment, the dipole moment, and the quadrupole moment, of this configuration of four charges.