Homework No. 02 (2019 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Friday, 2019 Aug 30, 2:00 PM, in class

1. (20 points.) Verify the following identities:

$$\nabla r = \frac{\mathbf{r}}{r} = \hat{\mathbf{r}},$$
 (1a)
 $\nabla \mathbf{r} = \mathbf{1}.$ (1b)

$$\nabla \mathbf{r} = \mathbf{1}.\tag{1b}$$

Further, show that

$$\nabla \cdot \mathbf{r} = 3,\tag{2a}$$

$$\nabla \times \mathbf{r} = 0. \tag{2b}$$

Here r is the magnitude of the position vector \mathbf{r} , and $\hat{\mathbf{r}}$ is the unit vector pointing in the direction of \mathbf{r} .

2. (20 points.) (Based on Problem 1.13, Griffiths 4th edition.) Show that

$$\nabla r^2 = 2\mathbf{r}.\tag{3}$$

Then evaluate ∇r^3 . Show that

$$\nabla \frac{1}{r} = -\frac{\hat{\mathbf{r}}}{r^2}.\tag{4}$$

Then evaluate $\nabla(1/r^2)$.

3. (20 points.) Use index notation or dyadic notation to show that

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}, \tag{5a}$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = (\nabla \times \mathbf{A}) \cdot \mathbf{B} - \mathbf{A} \cdot (\nabla \times \mathbf{B}). \tag{5b}$$