

Homework No. 01 (Fall 2019)

PHYS 500A: Mathematical Methods

Due date: Tuesday, 2019 Aug 27, 4.00pm

1. (10 points.) In spherical polar coordinates a point is coordinated by the intersection of family of spheres, cones, and half-planes, given by

$$r = \sqrt{x^2 + y^2 + z^2}, \quad (1a)$$

$$\theta = \tan^{-1} \sqrt{\frac{x^2 + y^2}{z^2}}, \quad (1b)$$

$$\phi = \tan^{-1} \frac{y}{x}, \quad (1c)$$

respectively. Show that the gradient of these surfaces are given by

$$\nabla r = \hat{\mathbf{r}}, \quad \hat{\mathbf{r}} = \sin \theta \cos \phi \hat{\mathbf{i}} + \sin \theta \sin \phi \hat{\mathbf{j}} + \cos \theta \hat{\mathbf{k}}, \quad (2a)$$

$$\nabla \theta = \hat{\boldsymbol{\theta}} \frac{1}{r}, \quad \hat{\boldsymbol{\theta}} = \cos \theta \cos \phi \hat{\mathbf{i}} + \cos \theta \sin \phi \hat{\mathbf{j}} - \sin \theta \hat{\mathbf{k}}, \quad (2b)$$

$$\nabla \phi = \hat{\boldsymbol{\phi}} \frac{1}{r \sin \theta}, \quad \hat{\boldsymbol{\phi}} = -\sin \phi \hat{\mathbf{i}} + \cos \phi \hat{\mathbf{j}}, \quad (2c)$$

which are normal to the respective surfaces. Sketch the surfaces and the corresponding normal vectors. This illustrates that $\nabla(\text{surface})$ is a vector (field) normal to the surface.