Midterm Exam No. 01 (Fall 2019)

PHYS 520A: Electromagnetic Theory I

Date: 2019 Sep 27

1. (**20 points.**) When magnetic charges ρ_m and magnetic currents $\mathbf{j}_m$ are permitted, in addition to electric charges ρ_e and electric currents $\mathbf{j}_e$, the Maxwell equations are

$$\boldsymbol{\nabla} \cdot \mathbf{D} = \rho_e, \tag{1a}$$

$$\boldsymbol{\nabla} \cdot \mathbf{B} = \rho_m, \tag{1b}$$

$$-\boldsymbol{\nabla} \times \mathbf{E} - \frac{\partial}{\partial t}\mathbf{B} = \mathbf{j}_m, \tag{1c}$$

$$\boldsymbol{\nabla} \times \mathbf{H} - \frac{\partial}{\partial t}\mathbf{D} = \mathbf{j}_e, \tag{1d}$$

where $\mathbf{D} = \varepsilon_0 \mathbf{E}$ and $\mathbf{B} = \mu_0 \mathbf{H}$. The Lorentz force density (force per unit volume) $\mathbf{f}$ is

$$\mathbf{f} = \rho_e \mathbf{E} + \mathbf{j}_e \times \mathbf{B} + \rho_m \mathbf{H} - \mathbf{j}_m \times \mathbf{D}. \tag{2}$$

In the absence of magnetic charges ρ_m and magnetic currents $\mathbf{j}_m$ we have the statement of conservation of electric charge given by

$$\frac{\partial}{\partial t}\rho_e + \boldsymbol{\nabla} \cdot \mathbf{j}_e = 0. \tag{3}$$

Repeat the derivation of the statement of conservation of electric charge in the presence of magnetic charges ρ_m and magnetic currents $\mathbf{j}_m$. In particular, inquire if the total electric charge in the universe is conserved in the presence of magnetic monopoles.

2. (**20 points.**) A monochromatic plane electromagnetic wave is described by electric and magnetic fields of the form

$$\mathbf{E} = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}, \tag{4a}$$

$$\mathbf{B} = \mathbf{B}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}, \tag{4b}$$

where $\mathbf{E}_0$ and $\mathbf{B}_0$ are constants. Assume no charges or currents.

   (a) Using Maxwell's equations show that

$$\mathbf{k} \cdot \mathbf{E} = 0, \tag{5a}$$

$$\mathbf{k} \cdot \mathbf{B} = 0, \tag{5b}$$

$$\mathbf{k} \times \mathbf{E} = \omega \mathbf{B}, \tag{5c}$$

$$\mathbf{k} \times \mathbf{B} = -\frac{\omega}{c^2}\mathbf{E}, \tag{5d}$$

   where $\varepsilon_0 \mu_0 = 1/c^2$.

(b) For non-trivial cases ($\mathbf{E}_0 \neq 0$ and $\mathbf{B}_0 \neq 0$), using Eqs. (5), show that we have

$$ck = \omega. \tag{6}$$

3. (**20 points.**) An electric dipole moment $\mathbf{p} = q\mathbf{a}$ consists of two equal and opposite charges q separated by a distance $\mathbf{a}$. A point electric dipole is an idealized limit of $a \to 0$, $q \to \infty$, keeping $p = aq$ fixed. The electric field of a point electric dipole moment is given by

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\Big[3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}\Big]. \tag{7}$$

The magnetic field $\mathbf{B} = 0$ everywhere. Show that the radial force F contributing to the electromagnetic stress, F/Area, on a sphere of radius r with the point dipole at the center of the sphere can be expressed in the form

$$F = \frac{c}{4\pi\varepsilon_0}\frac{p^2}{r^4}, \tag{8}$$

where c is a number. Find c.

4. (**20 points. Take home.**) Summarize Sec. III A of the article by Timothy H. Boyer titled '*Illustrations of Maxwell's term and the four conservation laws of electromagnetism*' in American Journal of Physics **87** (2019) 729. Interpret the results and answer whether the energy stored in the electromagnetic fields between the plates is increasing or decreasing with time. Verify if your answer is consistent with the direction of the flux of energy.