Midterm Exam No. 02 (Fall 2019)

PHYS 520A: Electromagnetic Theory I

Date: 2019 Nov 1

1. (**20 points.**) A simple model for susceptibility, suitable for insulators, is

$$\chi(\omega) = \frac{\omega_1}{\omega_0 - \omega} + i\,\pi\omega_1\delta(\omega - \omega_0), \tag{1}$$

   where ω_0 and ω_1 represent physical parameters of a material. Check if this model satisfies the Kramers-Kronig relation.

2. (**20 points.**) Consider a uniformly polarized half-slab, that occupies half of space, and has the direction of its polarization transverse to the direction $\hat{\mathbf{z}}$ normal to the surface of slab, described by

$$\mathbf{P}(\mathbf{r}) = \sigma\,\hat{\mathbf{x}}\,\theta(-z), \tag{2}$$

   where σ is the polarization per unit area of the slab and θ is the Heaviside step function. Determine the electric field, inside and outside the slab?

3. (**20 points.**) Consider a uniformly polarized sphere of radius R described by

$$\mathbf{P}(\mathbf{r}) = \alpha\,\mathbf{r}\,\theta(R - r), \tag{3}$$

   where α is a constant and θ is the Heaviside step function. Determine the electric field, inside and outside the sphere?

4. (**20 points. Take Home.**) Consider a solid sphere of radius R with uniform permanent polarization

$$\mathbf{P}(\mathbf{r}, t) = \mathbf{P}_0\,\theta(R - r), \tag{4}$$

   where $\mathbf{P}_0$ is a uniform vector, $\theta(x)$ is the Heaviside step function, and $r^2 = x^2 + y^2 + z^2$.

   (a) Determine the electric field and compare this with the electric field due to a point dipole. Plot the electric field, both outside and inside the sphere.

   (b) We have the constituent relation

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}. \tag{5}$$

   Determine the expression for $\mathbf{D}$. Draw the field lines of $\mathbf{D}$, both outside and inside the sphere. How is this different from the field lines of the electric field.

5. (**20 points. Take Home.**) The constitutive relations in a nondispersive media are

$$\mathbf{D} = \varepsilon \mathbf{E}, \tag{6a}$$

$$\mathbf{B} = \mu \mathbf{H}, \tag{6b}$$

where ε and μ are constants. The ratio of speed of light in vacuum c to speed of light in the medium v is the refractive index of the medium

$$n = \frac{c}{v} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}. \tag{7}$$

The theory of relativity states that velocity of energy flow can not be larger than the speed of light in vacuum. Thus, $n > 1$. Let $\mu = 1$. Consider the dielectric model

$$\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + \frac{\omega_p^2}{\omega_0^2 - i\omega\gamma - \omega^2}. \tag{8}$$

This is a complex number, which means a complex velocity of propagation v and a complex index of refraction

$$n = n_r + in_i = \frac{c}{v} = \sqrt{\frac{\varepsilon(\omega)}{\varepsilon_0}}. \tag{9}$$

A complex refractive index signifies that the propagation is accompanied by absorption

$$e^{-i\omega\left(t-\frac{x}{v}\right)} = e^{-i\omega\left(t-n\frac{x}{c}\right)} = e^{-n_i\frac{\omega}{c}x}e^{-i\omega\left(t-n_r\frac{x}{c}\right)}. \tag{10}$$

Thus, c/n_r plays the role of phase velocity and $n_i\omega/c$ is a coefficient of absorption. Plot n_r as a function of ω and verify that it crosses the line $n = 1$ near $\omega = \omega_0$. Thus, apparently, signal in a dispersive medium violates causality. This contradiction was resolved by Sommerfeld and Brillouin in 1914. Translated versions of their papers have been published in a book titled 'Wave Propagation and Group Velocity' by Brillouin in 1960. The book is available at https://archive.org. Very briefly present the resolution here.