Homework No. 02 (Fall 2019)

PHYS 520A: Electromagnetic Theory I

Due date: Friday, 2019 Sep 6, 4.00pm

1. (**20 points.**) Show that
$$\boldsymbol{\nabla}(\hat{\mathbf{r}}\cdot\mathbf{a}) = -\frac{1}{r}\,\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\mathbf{a}) \tag{1}$$
for a uniform (homogeneous in space) vector $\mathbf{a}$.

2. (**20 points.**) Evaluate the number evaluated by the expression
$$\frac{1}{2}\left[\hat{\boldsymbol{\rho}}\frac{\partial}{\partial\rho} + \hat{\boldsymbol{\phi}}\frac{1}{\rho}\frac{\partial}{\partial\phi}\right]\cdot(\rho\hat{\boldsymbol{\rho}}), \tag{2}$$
where $\hat{\boldsymbol{\rho}}$ and $\hat{\boldsymbol{\phi}}$ are the unit vectors for cylindrical coordinates (ρ,ϕ) given by
$$\hat{\boldsymbol{\rho}} = \cos\phi\,\hat{\mathbf{i}} + \sin\phi\,\hat{\mathbf{j}}, \tag{3}$$
$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{i}} + \cos\phi\,\hat{\mathbf{j}}. \tag{4}$$

3. (**10 points.**) Show that
$$\int_{-\infty}^{\infty} dx\, f(x)\,\delta(x^2-a^2) = \begin{cases} \frac{f(a)}{|a|}, & \text{if } f(a) \text{ is an even function,} \\ 0, & \text{if } f(a) \text{ is an odd function.} \end{cases} \tag{5}$$

4. (**10 points.**) An (idealized) infinitely long wire, (on the z-axis with infinitesimally small cross sectional area,) carrying a current I can be mathematically represented by the current density
$$\mathbf{J}(\mathbf{x}) = \hat{\mathbf{z}}\,I\,\delta(x)\delta(y). \tag{6}$$
A similar idealized wire forms a circular loop and is placed on the xy-plane with the center of the circular loop at the origin. Write down the current density of the circular loop carrying current I.