Homework No. 06 (Fall 2019)

PHYS 520A: Electromagnetic Theory I

Due date: Monday, 2019 Oct 28, 4.00pm

1. (**20 points.**) The response of a material to an electric field, in a particular model, is described by the susceptibility function

$$\chi(\omega) = \frac{\omega_p^2}{\omega_0^2 - i\omega\gamma}, \tag{1}$$

where ω_p, ω_0, and γ are material dependent parameters, and ω is the frequency of oscillation of the electric field.

   (a) $[\mathrm{Re}\chi(\omega)]$ is a measure of the square of the refractive index. Plot $[\mathrm{Re}\chi(\omega)]$ as a function of ω.

   (b) $[\mathrm{Im}\chi(\omega)]$ is a measure of absorption of light. Plot $[\mathrm{Im}\chi(\omega)]$ as a function of ω.

2. (**20 points.**) A simple model for susceptibility is

$$\chi(\omega) = \frac{\omega_1}{\omega_0 - \omega} + i\,\pi\omega_1\delta(\omega - \omega_0), \tag{2}$$

where ω_0 and ω_1 represent physical parameters of a material.

   (a) Note that

$$[\mathrm{Re}\chi(\omega)] = \frac{\omega_1}{\omega_0 - \omega} \qquad \text{and} \qquad [\mathrm{Im}\chi(\omega)] = \pi\omega_1\delta(\omega - \omega_0). \tag{3}$$

   (b) Plot $[\mathrm{Re}\chi(\omega)]$ and $[\mathrm{Im}\chi(\omega)]$ with respect to ω.

   (c) Evaluate the right hand side of the Kramers-Kronig relation

$$[\mathrm{Re}\chi(\omega)] = \lim_{\delta\to 0+} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} [\mathrm{Im}\chi(\omega')]\, 2\mathrm{Re}\left\{\frac{1}{\omega' - (\omega + i\delta)}\right\} \tag{4}$$

   for this simple model.