# Final Exam (2020 Spring)

## PHYS 301: THEORETICAL METHODS IN PHYSICS

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Instruction: You are allowed to refer to any resource of your choice. Choose to be honest and fair to your peers. After completing the exam, scan your work and submit it as a single PDF file to `shajesh@siu.edu`. This exam is due on the official date of the exam.

1. (**20 points.**) Find the real and imaginary part of the function
$$f = \cos z. \tag{1}$$
Here $z$ represents a complex number.

2. (**20 points.**) Evaluate the integral
$$\int_{-1}^{1} dx\, \delta(1-4x)\Big[64x^2+4x-1\Big]. \tag{2}$$

3. (**20 points.**) The Fourier space is spanned by the Fourier eigenfunctions
$$e^{im\phi}, \qquad m=0,\pm1,\pm2,\ldots, \qquad 0\le\phi<2\pi. \tag{3}$$
An arbitrary function $f(\phi)$ has the Fourier series representation
$$f(\phi)=\frac{1}{2\pi}\sum_{m=-\infty}^{\infty} a_m e^{im\phi}, \tag{4}$$
where $e^{im\phi}$ are the Fourier eigenfunctions and $a_m$ are the respective Fourier components. Determine all the Fourier components $a_m$ for the function $\sin^2\phi$.

4. (**20 points.**) The Legendre polynomials of order $l$ satisfy the recurrence relation
$$(2l+1)xP_l(x)=(l+1)P_{l+1}(x)+lP_{l-1}(x), \qquad l=1,2,3,\ldots. \tag{5}$$
Given,
$$P_0(x)=1, \tag{6a}$$
$$P_1(x)=x. \tag{6b}$$
Derive the explicit expression for $P_4(x)$.

5. (**20 points.**) Plot the Legendre polynomial $P_4(x)$. Note that $-1<x<1$. Also, determine $P_4(0)$.