Homework No. 06 (2020 Spring)

PHYS 301: THEORETICAL METHODS IN PHYSICS

Department of Physics, Southern Illinois University–Carbondale

Due date: Monday, 2020 Feb 24, 9:00 AM, in class

0. Problems 2 and 3 are to be submitted for assessment. Rest are for practice.

0. Keywords: Delta function, Heaviside step function.

1. (**70 points.**) (Based on problem 1.44,45/1.43,44 Griffiths 4th/3rd edition.) Evaluate the following integrals:

$$\int_2^6 dx \left[3x^2 - 2x - 3\right] \delta(x-3) = \tag{1a}$$

$$\int_{-7}^7 dx \, \sin x \, \delta(x-\pi) = \tag{1b}$$

$$\int_0^3 dx \, x^3 \, \delta(x+1) = \tag{1c}$$

$$\int_{-2}^2 dx \, [3x+3] \, \delta(3x) = \tag{1d}$$

$$\int_{-2}^2 dx \, [3x+3] \, \delta(-3x) = \tag{1e}$$

$$\int_0^2 dx \, [3x+3] \, \delta(1-x) = \tag{1f}$$

$$\int_{-1}^1 dx \, 9x^3 \, \delta(3x+1) = \tag{1g}$$

2. (**10 points.**) The distance between two points $\mathbf{r}$ and $\mathbf{r}'$ in rectangular coordinates is explicitly given by

$$|\mathbf{r} - \mathbf{r}'| = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}. \tag{2}$$

The charge density of a charge q at the origin is described in terms of delta functions as

$$\rho(\mathbf{r}) = q\delta(x)\delta(y)\delta(z). \tag{3}$$

Evaluate the electric potential at the observation point $\mathbf{r}$, due to a point charge q placed at source point $\mathbf{r}'$, using

$$\phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}, \tag{4}$$

where $\int d^3r' = \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dz'$. That is, evaluate the three integrals in

$$\phi(\mathbf{r}) = \frac{q}{4\pi\varepsilon_0} \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dz' \frac{\delta(x')\delta(y')\delta(z')}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}. \tag{5}$$

3. (**20 points.**) Evaluate

$$\frac{d}{dz}|z| \tag{6}$$

and

$$\frac{d^2}{dz^2}|z|, \tag{7}$$

in terms of the Heaviside step function

$$\theta(z) = \begin{cases} 0, & z < 0, \\ 1, & z > 0, \end{cases} \tag{8}$$

and the delta function.