Homework No. 08 (2020 Spring)

PHYS 301: THEORETICAL METHODS IN PHYSICS

Department of Physics, Southern Illinois University–Carbondale

Due date: Monday, 2020 Mar 16, 9:00 AM, in class

0. Problems 2 and 3 are to be submitted for assessment. Rest are for practice.

0. Keywords: Properties of Pauli matrices.

1. (**30 points.**) The Pauli matrices are traceless Hermitian matrices that satisfy

$$\sigma_i \sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k, \tag{1}$$

where i, j, are either $1, 2$, or 3. Show that these correspond to the following nine explicit equations.

$$\sigma_1^2 = 1, \tag{2a}$$
$$\sigma_2\sigma_1 = -i\sigma_3, \tag{2b}$$
$$\sigma_3\sigma_1 = i\sigma_2, \tag{2c}$$
$$\sigma_1\sigma_2 = i\sigma_3, \tag{2d}$$
$$\sigma_2^2 = 1, \tag{2e}$$
$$\sigma_3\sigma_2 = -i\sigma_1, \tag{2f}$$
$$\sigma_1\sigma_3 = -i\sigma_2, \tag{2g}$$
$$\sigma_2\sigma_3 = i\sigma_1, \tag{2h}$$
$$\sigma_3^2 = 1. \tag{2i}$$

2. (**30 points.**) Pauli matrices are Hermitian. That is,

$$\sigma_i^\dagger = \sigma_i. \tag{3}$$

Evaluate

$$(\sigma_i\sigma_j)^\dagger. \tag{4}$$

3. (**30 points.**) Pauli matrices are traceless. That is,

$$\mathrm{tr}(\sigma_i) = 0. \tag{5}$$

Evaluate

$$\mathrm{tr}(\sigma_i\sigma_j). \tag{6}$$

4. (**30 points.**) Evaluate the inverse

$$(\sigma_i)^{-1}. \tag{7}$$

Then, evaluate

$$(\sigma_i\sigma_j)^{-1}. \tag{8}$$