Midterm Exam No. 03 (2020 Spring)

PHYS 420: Electricity and Magnetism II

Department of Physics, Southern Illinois University–Carbondale
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Instruction: You are allowed to refer to any resource of your choice. Choose to be honest and fair to your peers. After completing the exam, scan your work and submit it as a single PDF file to `shajesh@siu.edu`. This exam is due on the official date of the exam.

1. (**20 points.**) Lorentz transformation describing a boost in the x-direction is obtained using the matrix
$$L = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{1}$$
   (a) Evaluate the determinant of the matrix L.
   (b) Determine L^{-1}.

2. (**20 points.**) Determine the non-relativistic limit ($v \ll c$) of the energy-momentum relation
$$E^2 = p^2c^2 + m^2c^4. \tag{2}$$
Give physical interpretation for all the terms.

3. (**20 points.**) Prove that if p_μ is a time-like vector and $p_\mu s^\mu = 0$, then s^μ is necessarily space-like.

4. (**20 points.**) Starting from the relation
$$u_\alpha u^\alpha = -c^2 \tag{3}$$
obtain
$$\frac{d\gamma}{dt} = \left(\frac{\mathbf{v}\cdot\mathbf{a}}{c^2}\right)\gamma^3. \tag{4}$$

5. (**20 points.**) Neutral π meson decays into two photons. That is,
$$\pi^0 \to \gamma_1 + \gamma_2. \tag{5}$$
Energy-momentum conservation for the decay in the laboratory frame, in which the meson is not necessarily at rest, is given by
$$p_\pi^\alpha = p_1^\alpha + p_2^\alpha. \tag{6}$$

Or, more specifically,

$$\left(\frac{E_\pi}{c}, \mathbf{p}\right) = \left(\frac{E_1}{c}, \mathbf{p}_1\right) + \left(\frac{E_2}{c}, \mathbf{p}_2\right), \tag{7}$$

where E_π and $\mathbf{p}$ are the energy and momentum of neutral π meson, and E_i's and $\mathbf{p}_i$'s are the energies and momentums of the photons. Thus, derive the relation

$$m_\pi^2 c^4 = 2E_1 E_2 (1 - \cos\theta), \tag{8}$$

where m_π is the mass of neutral π meson, and θ is the angle between the directions of $\mathbf{p}_1$ and $\mathbf{p}_2$.