

Homework No. 05 (2020 Spring)

PHYS 420: ELECTRICITY AND MAGNETISM II

Department of Physics, Southern Illinois University–Carbondale

Due date: None

Let the unit vectors associated with curvilinear coordinates (ξ_1, ξ_2, ξ_3) be $(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3)$ and let (h_1, h_2, h_3) be the respective scale factors. The gradient operator in these coordinates has the form

$$\nabla = \hat{\mathbf{e}}_1 \frac{1}{h_1} \frac{\partial}{\partial \xi_1} + \hat{\mathbf{e}}_2 \frac{1}{h_2} \frac{\partial}{\partial \xi_2} + \hat{\mathbf{e}}_3 \frac{1}{h_3} \frac{\partial}{\partial \xi_3}. \quad (1)$$

A vector field $\mathbf{N}(\xi_1, \xi_2, \xi_3)$ in these coordinates has the form

$$\mathbf{N} = N_1(\xi_1, \xi_2, \xi_3) \hat{\mathbf{e}}_1 + N_2(\xi_1, \xi_2, \xi_3) \hat{\mathbf{e}}_2 + N_3(\xi_1, \xi_2, \xi_3) \hat{\mathbf{e}}_3. \quad (2)$$

The curl in these coordinates can be evaluated as the determinant

$$\nabla \times \mathbf{N} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{\mathbf{e}}_1 & h_2 \hat{\mathbf{e}}_2 & h_3 \hat{\mathbf{e}}_3 \\ \frac{\partial}{\partial \xi_1} & \frac{\partial}{\partial \xi_2} & \frac{\partial}{\partial \xi_3} \\ h_1 N_1 & h_2 N_2 & h_3 N_3 \end{vmatrix}. \quad (3)$$

1. (20 points.) Given

$$\mathbf{N} = \hat{\mathbf{z}} \ln \rho \quad (4)$$

and the gradient operator

$$\nabla = \hat{\boldsymbol{\rho}} \frac{\partial}{\partial \rho} + \hat{\boldsymbol{\phi}} \frac{1}{\rho} \frac{\partial}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial}{\partial z} \quad (5)$$

evaluate

$$\nabla \times \mathbf{N}. \quad (6)$$

2. (20 points.) Given

$$\mathbf{N} = \frac{\hat{\boldsymbol{\phi}}}{\rho} \quad (7)$$

and the gradient operator

$$\nabla = \hat{\boldsymbol{\rho}} \frac{\partial}{\partial \rho} + \hat{\boldsymbol{\phi}} \frac{1}{\rho} \frac{\partial}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial}{\partial z} \quad (8)$$

evaluate

$$\nabla \times \mathbf{N}. \quad (9)$$

3. **(20 points.)** Given

$$\mathbf{N} = \hat{\phi} \frac{\rho}{(\rho^2 + z^2)^{\frac{3}{2}}} \quad (10)$$

and the gradient operator

$$\nabla = \hat{\rho} \frac{\partial}{\partial \rho} + \hat{\phi} \frac{1}{\rho} \frac{\partial}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial}{\partial z} \quad (11)$$

evaluate

$$\nabla \times \mathbf{N}. \quad (12)$$

4. **(20 points.)** Given

$$\mathbf{N} = \hat{\phi} \frac{\sin \theta}{r^2} \quad (13)$$

and the gradient operator

$$\nabla = \hat{\mathbf{r}} \frac{\partial}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \quad (14)$$

evaluate

$$\nabla \times \mathbf{N}. \quad (15)$$