

Homework No. 08 (2020 Spring)

PHYS 420: ELECTRICITY AND MAGNETISM II

Department of Physics, Southern Illinois University–Carbondale

Due date: Friday, 2019 Mar 20, 2:00 PM, in class

0. **(0 points.)** Keywords for finding resource materials: Special relativity, Lorentz transformation, time dilation, length contraction, relativistic velocity addition formula.
0. Problems 3 is to be submitted for assessment. Rest are for practice.
1. **(20 points.)** The Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}, \quad \beta = \frac{v}{c}. \quad (1)$$

- (a) Evaluate γ for $v = 30 \text{ m/s}$ ($\sim 70 \text{ miles/hour}$).
- (b) Evaluate γ for $v = 3c/5$.
2. **(20 points.)** Lorentz transformation describing a boost in the x -direction is obtained using the matrix

$$L = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (2)$$

- (a) Show that the determinant of the matrix L is 1.
- (b) Determine L^{-1} .
3. **(60 points.)** The Poincaré formula for the addition of (parallel) velocities is

$$v = \frac{v_a + v_b}{1 + \frac{v_a v_b}{c^2}}, \quad (3)$$

where v_a and v_b are velocities and c is speed of light in vacuum. Jerzy Kocik, from the department of Mathematics in SIUC, has invented a geometric diagram that allows one to visualize the Poincaré formula. (Refer [1].) An interactive applet for exploring velocity addition is available at Kocik's web page [2]. (For the following assume that the Poincaré formula holds for all speeds, subluminal ($v_i < c$), superluminal ($v_i > c$), and speed of light.)

- (a) Analyse what is obtained if you add two subluminal speeds?

- (b) Analyse what is obtained if you add a subluminal speed to speed of light?
- (c) Analyse what is obtained if you add a subluminal speed to a superluminal speed?
- (d) Analyse what is obtained if you add speed of light to another speed of light?
- (e) Analyse what is obtained if you add a superluminal speed to speed of light?
- (f) Analyse what is obtained if you add two superluminal speeds?

4. **(30 points.)** Let

$$\tanh \theta = \beta, \quad (4)$$

where $\beta = v/c$. Addition of (parallel) velocities in terms of the parameter θ obeys the arithmetic addition

$$\theta = \theta_a + \theta_b. \quad (5)$$

- (a) Invert the expression in Eq. (4) to find the explicit form of θ in terms of β as a logarithm.
- (b) Show that Eq. (5) leads to the relation

$$\left(\frac{1 + \beta}{1 - \beta} \right) = \left(\frac{1 + \beta_a}{1 - \beta_a} \right) \left(\frac{1 + \beta_b}{1 - \beta_b} \right). \quad (6)$$

- (c) Using Eq. (6) derive the Poincaré formula for the addition of (parallel) velocities.

References

- [1] J. Kocik. Geometric diagram for relativistic addition of velocities. *Am. J. Phys.*, 80:737–739, August 2012.
- [2] J. Kocik. An interactive applet for exploring relativistic velocity addition. <http://www.mathoutlet.com/2016/08/relativistic-composition-of-velocities.html>.