Final Exam (2021 Spring)

PHYS 420: Electricity and Magnetism II

Department of Physics, Southern Illinois University–Carbondale

2021 May 3

1. (**20 points.**) Determine the total magnetic dipole moment for the following configuration. The current in the loop is I and each fold in the loop is of length a.

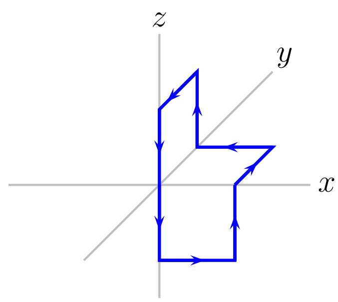

Figure 1: Problem 1

2. (**20 points.**) Find a four-vector of the form

$$q^\alpha = (q^0, 0, 0, q^3) \tag{1}$$

that is perpendicular to

$$p^\alpha = (1, 0, 0, 2), \tag{2}$$

3. (**20 points.**) Using the identity

$$\delta(F(x)) = \sum_r \frac{\delta(x - a_r)}{\left| \frac{dF}{dx}\Big|_{x=a_r} \right|}, \tag{3}$$

where the sum on r runs over the roots a_r of the equation $F(x) = 0$, determine the associated identity for

$$\delta(x^2 + 4x - 21). \tag{4}$$

4. (**20 points.**) A charge particle with charge q moves on the z-axis with constant speed v, $\beta = v/c$. The electric and magnetic field generated by this charged particle is given by

$$\mathbf{E}(\mathbf{r}, t) = (1 - \beta^2) \frac{q}{4\pi\varepsilon_0} \frac{x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + (z - vt)\hat{\mathbf{k}}}{[(x^2 + y^2)(1 - \beta^2) + (z - vt)^2]^{\frac{3}{2}}}, \tag{5a}$$

$$c\mathbf{B}(\mathbf{r}, t) = \beta(1 - \beta^2) \frac{q}{4\pi\varepsilon_0} \frac{-y\hat{\mathbf{i}} + x\hat{\mathbf{j}}}{[(x^2 + y^2)(1 - \beta^2) + (z - vt)^2]^{\frac{3}{2}}}. \tag{5b}$$

Illustrate how the above fields differ from those of a charge particle at rest. Evaluate the electromagnetic momentum density for this configuration by evaluating

$$\mathbf{G}(\mathbf{r},t) = \varepsilon_0 \mathbf{E}(\mathbf{r},t) \times \mathbf{B}(\mathbf{r},t). \tag{6}$$

5. (**20 points.**) Given the retarded time

$$t_r = t - \frac{r}{c} + \frac{\hat{\mathbf{r}} \cdot \mathbf{r}'}{c}, \tag{7}$$

evaluate

$$\boldsymbol{\nabla} t_r \tag{8}$$

and

$$\boldsymbol{\nabla}' t_r. \tag{9}$$