Midterm Exam No. 01 (2021 Spring)

PHYS 420: Electricity and Magnetism II

Department of Physics, Southern Illinois University–Carbondale
Due date: Monday, 2021 Feb 22, 2:00 PM

1. (**20 points.**) A charged particle in a magnetic field goes in circles (or in helices). Recall that positron is the antiparticle of electron. Describe the motion of a positron in a magnetic field, and contrast it to that of an electron in a magnetic field. How will the ionization track of electron and positron differ in a bubble chamber? For example, refer to the picture at 34:21 minute in the lecture by Frank Close, part of

   Christmas Lectures, 1993.

2. (**20 points.**) Consider a straight wire of radius a carrying current I described using the current density
$$\mathbf{J}(\mathbf{r}) = \hat{\mathbf{z}}\,\frac{C}{\rho}\,e^{-\lambda\rho}\,\theta(a-\rho), \tag{1}$$
where $\theta(x) = 1$ for $x > 1$ and zero otherwise.
   (a) Find C in terms of the current I.
   (b) Find the magnetic field inside and outside the wire.
   (c) Plot the magnetic field as a function of ρ.

3. (**20 points.**) A circular wire carrying current I forms a loop of radius a and is described by current density
$$\mathbf{j}(\mathbf{r}') = \hat{\boldsymbol{\phi}}' I \delta(z')\delta(\rho' - a). \tag{2}$$
Determine the magnetic vector potential using
$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int d^3r' \frac{\mathbf{j}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \tag{3}$$
on the axis of the circular wire at $\mathbf{r} = z\,\hat{\mathbf{k}}$. Determine the magnetic field using
$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int d^3r' \mathbf{j}(\mathbf{r}') \times \frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \tag{4}$$
on the axis of the circular wire at $\mathbf{r} = z\,\hat{\mathbf{k}}$.

4. (**20 points.**) The magnetic field $\mathbf{B}$ is determined using the vector potential $\mathbf{A}$ by the relation
$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}. \tag{5}$$
Determine the vector potential for a uniform magnetic field pointing in the $\hat{\mathbf{z}}$ direction. Is this a unique construction.