Homework No. 02 (2021 Spring)

PHYS 420: ELECTRICITY AND MAGNETISM II

Department of Physics, Southern Illinois University–Carbondale

Due date: Friday, 2021 Feb 5, 2:00 PM

0. Keywords: Magnetostatics (Chap. 5, Griffiths 4th edition), Ampere's law (Sec. 5.2, Griffiths 4th edition), Bio-Savart (Sec. 5.2, Griffiths 4th edition).

0. Problems 1 and 4 are to be submitted for assessment. Rest are for practice.

1. (**20 points.**) A steady current I flows down an infinitely long cylindrical wire of radius a. Using Ampère's law find the magnetic field, both inside and outside the wire, if the current is uniformly distributed over the outside surface of the wire.

2. (**30 points.**) Using Ampere's law determine the magnetic field inside and outside a solenoid of radius R and of infinite extent in the directions of its symmetry axis.

3. (**30 points.**) A solenoid has the geometry of a right circular cylinder of radius a and height extending to infinity on both ends. Using Ampere's law show that the magnetic field is uniform inside the solenoid and zero outside the solenoid. How does this result change for a solenoid of arbitrary cross section. Refer literature. The results need not be reproduced here.

4. (**20 points.**) An infinitely long wire of circular cross section radius a carries a steady current I. Another wire, in the form of a cylindrical shell and concentric to the first wire, has inner radius b and outer radius c, such that $a < b < c$. The region enclosed by $a < \rho < b$ and $c < \rho$ is empty space. The outer wire carries the same current I in the opposite direction. Let the direction of z-axis be along the wire.

   (a) Use Ampere's law to find the expression for magnetic field in the four regions, $\rho < a$, $a < \rho < b$, $b < \rho < c$, and $c < \rho$.

   (b) Plot the resulting magnetic field as a function of ρ.

5. (**30 points.**) Consider a straight wire of radius a carrying current I described using the current density

$$\mathbf{J}(\mathbf{r}) = \hat{\mathbf{z}}\,\frac{C}{\rho}\,e^{-\lambda\rho}\,\theta(a-\rho), \tag{1}$$

   where $\theta(x) = 1$ for $x > 1$ and zero otherwise.

   (a) Find C in terms of the current I.

   (b) Find the magnetic field inside and outside the wire.

(c) Plot the magnetic field as a function of ρ.

6. (**30 points.**) A steady current I flows down a long cylindrical wire of radius a. The current density in the wire is described by, $n > 0$,

$$\mathbf{J}(\mathbf{r}) = \hat{\mathbf{z}}\, \frac{I}{2\pi a^2}\, (n+2) \left(\frac{\rho}{a}\right)^n \theta(a-\rho). \tag{2}$$

(a) Show that, indeed,

$$\int_S d\mathbf{S} \cdot \mathbf{J}(\mathbf{r}) = I. \tag{3}$$

(b) Using Ampere's law show that the magnetic field inside and outside the cylinder is given by

$$\mathbf{B}(\mathbf{r}) = \begin{cases} \dfrac{\mu_0}{4\pi} \dfrac{2I}{\rho} \left(\dfrac{\rho}{a}\right)^{n+2} \hat{\boldsymbol{\phi}} & \rho < a, \\[2ex] \dfrac{\mu_0}{4\pi} \dfrac{2I}{\rho}\, \hat{\boldsymbol{\phi}} & \rho > a. \end{cases} \tag{4}$$

(c) Plot the magnitude of the magnetic field as a function of ρ.