Homework No. 07 (2021 Spring)

PHYS 420: ELECTRICITY AND MAGNETISM II

Department of Physics, Southern Illinois University–Carbondale
Due date: Friday, 2021 Mar 26, 2:00 PM

0. (**0 points.**) Keywords for finding resource materials: Special relativity, Lorentz transformation, time dilation, length contraction, relativistic velocity addition formula.

0. Problems 1, 2, and 3 are to be submitted for assessment. Rest are for practice.

1. (**20 points.**) The Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \qquad \beta = \frac{v}{c}. \tag{1}$$

   (a) Evaluate γ for $v = 30\,\text{m/s}$ ($\sim 70\,\text{miles/hour}$).

   (b) Evaluate γ for $v = 3c/5$.

2. (**20 points.**) Lorentz transformation describing a boost in the x-direction is obtained using the matrix

$$L = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{2}$$

   (a) Show that the determinant of the matrix L is 1.

   (b) Determine L^{-1}.

3. (**60 points.**) The Poincaré formula for the addition of (parallel) velocities is

$$v = \frac{v_a + v_b}{1 + \dfrac{v_a v_b}{c^2}}, \tag{3}$$

   where v_a and v_b are velocities and c is speed of light in vacuum. Jerzy Kocik, from the department of Mathematics in SIUC, has invented a geometric diagram that allows one to visualize the Poincaré formula. (Refer [1].) An interactive applet for exploring velocity addition is available at Kocik's web page [2]. (For the following assume that the Poincaré formula holds for all speeds, subluminal ($v_i < c$), superluminal ($v_i > c$), and speed of light.)

   (a) Analyse what is obtained if you add two subluminal speeds?

(b) Analyse what is obtained if you add a subluminal speed to speed of light?

(c) Analyse what is obtained if you add a subluminal speed to a superluminal speed?

(d) Analyse what is obtained if you add speed of light to another speed of light?

(e) Analyse what is obtained if you add a superluminal speed to speed of light?

(f) Analyse what is obtained if you add two superluminal speeds?

4. (**30 points.**) Let

$$\tanh\theta = \beta, \tag{4}$$

where $\beta = v/c$. Addition of (parallel) velocities in terms of the parameter θ obeys the arithmatic addition

$$\theta = \theta_a + \theta_b. \tag{5}$$

(a) Invert the expression in Eq. (4) to find the explicit form of θ in terms of β as a logarithm.

(b) Show that Eq. (5) leads to the relation

$$\left(\frac{1+\beta}{1-\beta}\right) = \left(\frac{1+\beta_a}{1-\beta_a}\right)\left(\frac{1+\beta_b}{1-\beta_b}\right). \tag{6}$$

(c) Using Eq. (6) derive the Poincaré formula for the addition of (parallel) velocities.

References

[1] J. Kocik. Geometric diagram for relativistic addition of velocities. *Am. J. Phys.*, 80:737–739, August 2012.

[2] J. Kocik. An interactive applet for exploring relativistic velocity addition. http://www.mathoutlet.com/2016/08/relativistic-composition-of-velocities.html.