Homework No. 08 (2021 Spring)

PHYS 420: ELECTRICITY AND MAGNETISM II

Department of Physics, Southern Illinois University–Carbondale

Due date: Monday, 2021 Apr 5, 2:00 PM

1. (**20 points.**) Determine the non-relativistic limit ($v \ll c$) of the energy-momentum relation

$$E^2 = p^2c^2 + m^2c^4. \tag{1}$$

   Give physical interpretation for all the terms.

2. (**20 points.**) Prove that if p_μ is a time-like vector and $p_\mu s^\mu = 0$, then s^μ is necessarily space-like.

3. (**20 points.**) Neutral π meson decays into two photons. That is,

$$\pi^0 \to \gamma_1 + \gamma_2. \tag{2}$$

   Energy-momentum conservation for the decay in the laboratory frame, in which the meson is not necessarily at rest, is given by

$$p_\pi^\alpha = p_1^\alpha + p_2^\alpha. \tag{3}$$

   Or, more specifically,

$$\left(\frac{E_\pi}{c}, \mathbf{p}\right) = \left(\frac{E_1}{c}, \mathbf{p}_1\right) + \left(\frac{E_2}{c}, \mathbf{p}_2\right), \tag{4}$$

   where E_π and $\mathbf{p}$ are the energy and momentum of neutral π meson, and E_i's and $\mathbf{p}_i$'s are the energies and momentums of the photons. Thus, derive the relation

$$m_\pi^2 c^4 = 2E_1E_2(1 - \cos\theta), \tag{5}$$

   where m_π is the mass of neutral π meson, and θ is the angle between the directions of $\mathbf{p}_1$ and $\mathbf{p}_2$.