

Homework No. 03 (2021 Spring)

PHYS 510: CLASSICAL MECHANICS

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2021 Feb 9, 9.30am

1. (20 points.) Given

$$F[u] = \int_{x_1}^{x_2} dx a(x) \frac{du(x)}{dx}. \quad (1)$$

Evaluate

$$\frac{\delta F}{\delta u(x)}. \quad (2)$$

2. (20 points.) (Gelfand and Fomin, Calculus of Variations.) Evaluate the functional derivatives of the following functionals, assuming no variation at the end points.

(a)

$$F[y] = \int_0^1 dx \frac{dy}{dx} \quad (3)$$

(b)

$$F[y] = \int_0^1 dx y \frac{dy}{dx} \quad (4)$$

(c)

$$F[y] = \int_0^1 dx xy \frac{dy}{dx} \quad (5)$$

(d)

$$F[y] = \int_a^b \frac{dx}{x^3} \left(\frac{dy}{dx} \right)^2 \quad (6)$$

3. (20 points.) The distance between two points in a plane is characterized by the infinitesimal statement

$$ds^2 = dx^2 + dy^2. \quad (7)$$

Let us prove the intuitively obvious statement that the curve of shortest distance going through two points (x_1, y_1) and (x_2, y_2) in a plane, the geodesics of a plane, is a straight line passing through the two points. This amounts to finding the extremal of the functional

$$l[y] = \int_{(x_1, y_1)}^{(x_2, y_2)} ds = \int_{(x_1, y_1)}^{(x_2, y_2)} dx \sqrt{1 + \left(\frac{dy}{dx} \right)^2}. \quad (8)$$

Using the fact that we require the curve to necessarily pass through the points (x_1, y_1) and (x_2, y_2) , show that

$$\frac{\delta l[y]}{\delta y(x)} = -\frac{d}{dx} \left[\frac{\frac{dy}{dx}}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} \right]. \quad (9)$$

Then, using the condition that the functional derivative is zero for the extremal curve, derive the equation of the geodesic to be given by

$$\frac{dy}{dx} = c, \quad (10)$$

where c is a constant. Identify this as the equation of a straight line in a plane. Find c .