Homework No. 08 (2021 Spring)

PHYS 510: CLASSICAL MECHANICS

Department of Physics, Southern Illinois University–Carbondale

Due date: Tuesday, 2021 Apr 6, 9.30am

1. (**30 points.**) The motion of a particle of mass m near the Earth's surface is described by
$$\frac{d}{dt}(mv) = -mg, \tag{1}$$
where $v = dz/dt$ is the velocity in the upward z direction.
   (a) Find the Lagrangian for this system that implies the equation of motion of Eq. (1) using Hamilton's principle of stationary action.
   (b) Determine the canonical momentum for this system
   (c) Determine the Hamilton $H(p, z)$ for this system.
   (d) Determine the Hamilton equations of motion.

2. (**30 points.**) The motion of a particle of mass m undergoing simple harmonic motion is described by
$$\frac{d}{dt}(mv) = -kx, \tag{2}$$
where $v = dx/dt$ is the velocity in the x direction.
   (a) Find the Lagrangian for this system that implies the equation of motion of Eq. (2) using Hamilton's principle of stationary action.
   (b) Determine the canonical momentum for this system
   (c) Determine the Hamilton $H(p, x)$ for this system.
   (d) Determine the Hamilton equations of motion.

3. (**30 points.**) A non-relativistic charged particle of charge q and mass m in the presence of a known electric and magnetic field is described by
$$\frac{d}{dt}(m\mathbf{v}) = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}. \tag{3}$$
   (a) Using
$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{4a}$$
$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}, \tag{4b}$$

find the Lagrangian for this system, that implies the equation of motion of Eq. (3), to be

$$L(\mathbf{x}, \mathbf{v}, t) = \frac{1}{2}mv^2 - q\phi + q\mathbf{v}\cdot\mathbf{A}, \tag{5}$$

using Hamilton's principle of stationary action.

(b) Determine the canonical momentum for this system

(c) Determine the Hamilton $H(\mathbf{x}, \mathbf{p}, t)$ for this system to be

$$H(\mathbf{x}, \mathbf{p}, t) = \frac{1}{2m}\left(\mathbf{p} - q\mathbf{A}\right)^2 + q\phi. \tag{6}$$

4. (**30 points.**) A relativistic charged particle of charge q and mass m in the presence of a known electric and magnetic field is described by

$$\frac{d}{dt}\left(\frac{m\mathbf{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}. \tag{7}$$

(a) Find the Lagrangian for this system, that implies the equation of motion of Eq. (7), to be

$$L(\mathbf{x}, \mathbf{v}, t) = -mc^2\sqrt{1-\frac{v^2}{c^2}} - q\phi + q\mathbf{v}\cdot\mathbf{A}, \tag{8}$$

using Hamilton's principle of stationary action.

(b) Determine the canonical momentum for this system

(c) Determine the Hamilton $H(\mathbf{r}, \mathbf{p})$ for this system to be

$$H(\mathbf{x}, \mathbf{p}, t) = \sqrt{m^2c^4 + \left(\mathbf{p} - q\mathbf{A}\right)^2 c^2} + q\phi. \tag{9}$$