Final Exam (2021 Spring)

PHYS 520B: ELECTROMAGNETIC THEORY

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1. (**20 points.**) Given the retarded time (in the far-field approximation)

$$t_r = t - \frac{r}{c} + \frac{\hat{\mathbf{r}} \cdot \mathbf{r}'}{c}, \tag{1}$$

evaluate

$$\boldsymbol{\nabla} t_r \tag{2}$$

and

$$\boldsymbol{\nabla}' t_r. \tag{3}$$

2. (**20 points.**) Using the identity

$$\delta(F(x)) = \sum_r \frac{\delta(x - a_r)}{\left| \frac{dF}{dx}\big|_{x=a_r} \right|}, \tag{4}$$

where the sum on r runs over the roots a_r of the equation $F(x) = 0$, determine the associated identity for

$$\delta\big((x-1)(x-2)(x-3)\big). \tag{5}$$

3. (**20 points.**) The magnetic field associated to radiation fields is given by

$$c\mathbf{B}(\mathbf{r}, t) = -\hat{\mathbf{r}} \times \frac{\mu_0 c}{4\pi} \frac{1}{r} \int d^3 r' \left\{ \frac{1}{c} \frac{\partial}{\partial t'} \mathbf{J}(\mathbf{r}', t') \right\}_{t'=t_r}, \tag{6}$$

where the contribution to the field comes at the retarded time

$$t_r = t - \frac{r}{c} + \hat{\mathbf{r}} \cdot \frac{\mathbf{r}'}{c}. \tag{7}$$

The associated electric field is given by

$$\mathbf{E}(\mathbf{r}, t) = -\hat{\mathbf{r}} \times c\mathbf{B}(\mathbf{r}, t), \tag{8}$$

and satisfies

$$c\mathbf{B}(\mathbf{r}, t) = \hat{\mathbf{r}} \times \mathbf{E}(\mathbf{r}, t). \tag{9}$$

For a simple antenna consisting of an infinitely thin conductor of length L carrying a time-dependent current, centered at the origin and placed on the z axis such that,

$$\mathbf{J}(\mathbf{r}', t') = \hat{\mathbf{z}}\, I_0 \sin \omega_0 t\, \delta(x')\delta(y')\theta(-L < 2z' < L), \tag{10}$$

where the function θ equals 1 when the argument is a true statement, and zero otherwise, evaluate the average power radiated per unit solid angle for the case $L \ll \lambda_0 = 2\pi c/\omega_0$.